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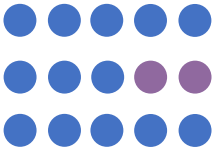
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**CHEMICAL  
STRUCTURES  
OF  
DIOXIN AND DIOXIN-LIKE  
COMPOUNDS AND  
TRANSITIONS  
TO LIVING ORGANISMS**

**BÖLÜM  
1**

**Cumali KESKİN<sup>1</sup>**

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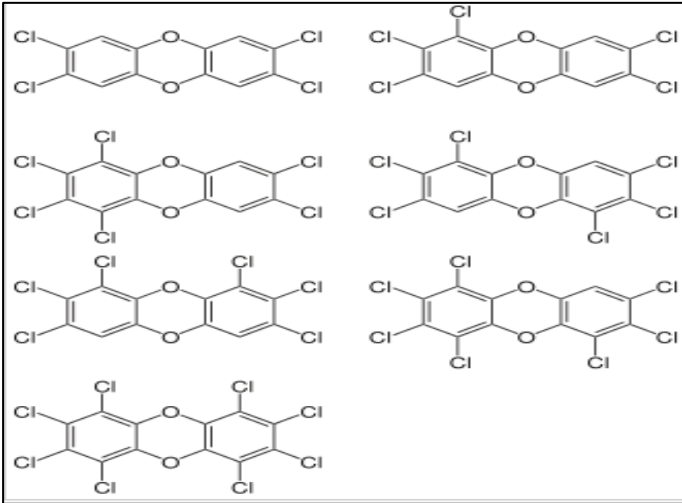
## 1. Introduction

Dioxin and dioxin-like compounds are toxic environmental pollutants that significantly threaten human and animal health, which can be found almost anywhere (White and Birnbaum, 2010). Dioxins are a large group of chemicals whose naturally absent and non-produced properties and toxicities are related to each other. The most known dioxins are polychlorinated  $p$ -dioxins (PCDD), polychlorinated dibenzofurans (PCDF) and polychlorobiphenyls (PCB). In addition, there are 75 different dioxins known and the most toxic among them is 2,3,7,8-tetrachlorodibenzo- $p$ -dioxin (TCDD) (Giesy and Kannan, 1998; Van den Berg, et al., 2006). The ability of dioxins to dissolve in water is limited, but due to their lipophilic properties, they can accumulate in foods and remain stable for quite a long time. Dioxins are found in high amounts in animal foods such as meat and meat products, milk and dairy products, and seafood, while they are found in quite low amounts in plant-derived foods. Therefore, the contact of dioxin with people is mostly through the food chain. Many negatives affecting health are observed in people, who exposed to dioxin, such as immunotoxicity, cardiotoxicity, birth defects, and developmental disorders in children (Ross, et al., 1996; Ni, et al., 2010; Waits and Nebert, 2011; Takahashi et al., 2017).

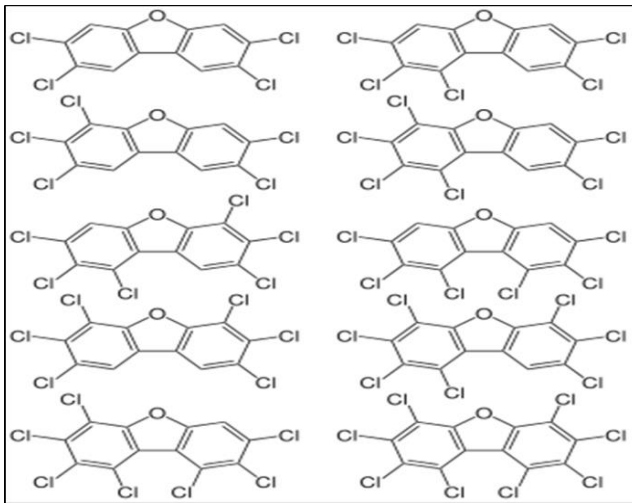
Dioxins are formed as a chemical contaminant in industrial production processes containing chlorine or bromine, and as a result of the combustion of an organic substance in the presence of chlorine. Structurally, some chemical materials are similar to dioxin and show “dioxin-like” behavior and toxicity. Among these chemicals can be count chlorodibenzofurans (polychlorinated benzofurans, PCDF or furans), polychlorinated diphenyl’s (PCBs) and naphthalenes. The chemical structures of PCDDs and PCDFs are given in

Figures 1 and 2. In addition, bromine-containing substances (similar chemicals in which chlorine and bromine are replaced) may have a dioxin-like toxic effect (Dale et al., 2004).

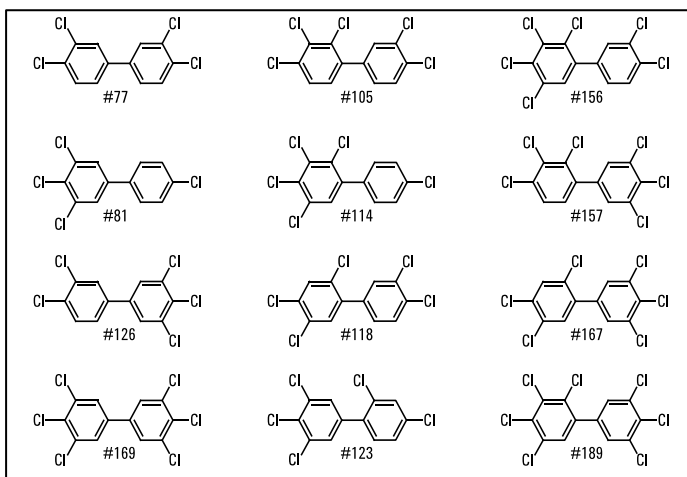
There are 135 different furan and 209 different PCB compounds detected. All dioxins, furans, and PCBs show different toxic properties from each other. Only 7 of 75 dioxin compounds are highly toxic and only 10 of 135 furans and 11 of 209 PCBs show dioxin-like toxicity. The structures of dioxin-like and non-dioxin-like PCBs are shown in detail in Figures 3 and 4. When the word dioxin is used, it is generally meant the common activity of these 28 kinds of substances. These 28 substances form similar toxic effects with similar mechanisms in the body. These substances bind to cellular macromolecules known as the "Ah receptor". The key factor in the toxic effect of these chemicals is the compatibility of the chemical substance molecule to the Ah receptor. Chemicals that bind tightly to the Ah receptor have greater toxicity than those that bind loosely. Among them, TCDD is a dioxin variety that is able to bind tightly with the Ah receptor and thus is the most toxic. Chemicals of the same size and form as TCDD, bind to the Ah receptor in the same manner, forming an effect close to it. Chemicals of different size and form as the structure cannot be properly bonded. Therefore, their toxicity is low or absent (Bannister and Safe, 1987; Pizarro-Aranguiz et al., 2015).



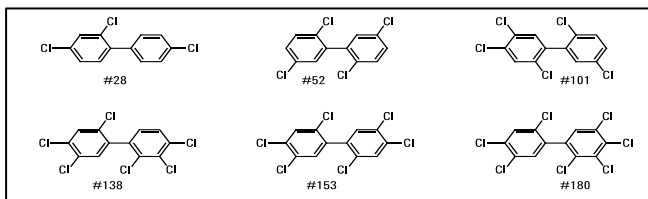
**Figure 1.** Chemical structures of 2,3,7,8-substitute PCDDs (Pohjanvirta and Tuomisto, 1994)



**Figure 2.** Chemical structures of 2,3,7,8-substitute PCDFs (Pohjanvirta and Tuomisto, 1994)



**Figure 3.** Chemical structures of dioxin-like-PCBs (*dl*-PCB)  
(Fürst et al., 2011)



**Figure 4.** Chemical structures of non-dioxin-like-PCBs (*ndl*-PCB)  
(Fürst et al., 2011)

## 1.1. Dioxin Resources

Parallel to industrial development, the 1960s and 1970s were the years when PCDD, PCDF, and dioxin-like PCBs began to appear frequently in the literature. The four main industrial activities play a role in the formation of dioxin: In the combustion units, the amount of dioxin that occurs as a result of the combustion event depends on the amount of chlorine in the burned waste. It

is also the second major source of dioxin in the paper industry because of the use of active chlorine to bleach the pulp. The third source of dioxin is produced as a chemical of chlorine-containing commercial products such as PVC, chlorinated solutions, painted stickers, and pesticides. The fourth source is industrial structures such as metal separation sites, refineries and cement kilns (Bavel et al., 2008; Lake et al., 2015).

Known dioxin sources can be listed as follows; Known PCDD / F production sources include (Arıkan et al., 2009); Natural events such as electricity generation and heating, motor vehicles, cigarette smoke, forest fires, volcanic eruption; animal feed, chemical production (pesticide, PVC and cosmetic industry, etc.), leather, textile and paper industry, uncontrolled combustion processes (combustion of general and medical waste, biomass combustion, etc.), lime, asphalt, cement production, ferrous and non-ferrous metal production, storage and deposition (deposition of waste oils, sludge treatment, etc.).

## **1.2. Toxic Equivalency of Dioxin**

Nutrients, water or soil can contain very different forms of dioxin, furans, and PCBs. Because some of the substances mentioned are more toxic than others, there needs to be an appropriate criterion for assessing the toxicity of all dioxin-like substances contained within the sample. According to the tests performed on the sample, it reflects the drawback that comes from only one type of dioxin, rather than the common effects of all dioxins and dioxin-like substances found. The EPA (Environmental Protection Agency) has developed a 2-stage method to identify the total toxic equivalence of the selected sample: In the first stage; with the help of an equation used the common toxic effects of all dioxin derivatives are converted to a unit. In the second stage; the common toxic effects of similar toxic substances in the same

sample are collected to express the total toxic equivalence. (Gallani et al., 2004; Akale and Tarekegn, 2017).

1)  $TEQ = \text{Dioxin concentration} \times \text{Toxicity factor}$

2) Total TEQ = Sum of all toxic TEQs in the sample

In the formula used for the first stage of the mentioned equations, it was determined as the toxic equivalency factor (TEF) of 2,3,7,8-TCDD which is the most toxic form of dioxin. The “toxicity factor” of each of the 17 types of dioxin or furan compounds, which are toxic, was determined by taking into account their relative toxicity according to TCDD (toxicity factors for PCBs have not yet been determined). The toxicity factor is different for each dioxin. TEQ (Total toxic equivalency) values determined by WHO are given in Tab

Analyzed substance		TEQ
2,3,7,8-TCDF		0.1
1,2,3,7,8-PeCDF		0.03
2,3,4,7,8-PeCDF		0.3
1,2,3,4,7,8-HxCDF		0.1
1,2,3,6,7,8-HxCDF		0.1
2,3,4,6,7,8-HxCDF		0.1
1,2,3,7,8,9-HxCDF		0.1
1,2,3,4,6,7,8-HpCDF		0.01
1,2,3,4,7,8,9-HpCDF	Dioxin	0.01
OCDF		0.0003
2,3,7,8-TCDD		1
1,2,3,7,8-PeCDD		1
1,2,3,4,7,8-HxCDD		0.1
1,2,3,6,7,8-HxCDD		0.1
1,2,3,7,8,9-HxCDD		0.1
1,2,3,4,6,7,8-HpCDD		0.01
OCDD		0.0003
PCB 123		0.0003
PCB 118		0.0003
PCB 114		0.0003
PCB 105	PCB	0.0003
PCB 167		0.0003
PCB 156		0.0003
PCB 157		0.0003
PCB 189		0.0003
PCB 81		0.0003
PCB 77	Dioxin-like PCBs (dl-PCB)	0.0001
PCB 126		0.1
PCB 169		0.03
PCB 28		1
PCB 52		1
PCB 101	Indicator PCB	1
PCB 153		1
PCB 138		1
PCB 180		1

Table 1. TEQ values determined by WHO for dioxin, dioxin-like dioxin and indicator dioxins (Van den Berg et al., 2006)

### 1.3.Dioxin in Foods

Dioxins are transported by air and stored in the solid or gas phase in water, soil, and plants; they accumulate more intensively, especially in animal tissues and soil (Abella, et al., 2015). The accumulation of dioxins in the animal body is due to the fact that animals consume more contaminated plants (Gallani et al., 2004; Abella, et al., 2015; Akale and Tarekegn, 2017). The largest source of dioxin-contaminated to water is regular storage and accounting for 75% of the total emission. However, the passage of dioxins into water is also realized by the discharge of wastewater and erosion. The United States Environmental Protection Agency (US EPA) has determined the maximum daily tolerable TCDD in drinking water to be 10 pg TEQ (Total Dioxin Toxic Equivalent) (Gallani et al., 2004; U.S Department Health and Human Service, 2009; Abella, et al., 2015; Akale and Tarekegn, 2017).

Dioxin is usually taken into the human body by consumption of animal products such as meat, milk, and fish (Trocino, et al., 2014). When 30 g of dairy products are consumed per day, the body is subject to intake of 2,3,7,8-TCDD at about 6 pg level. Because of TCDD is metabolized much more slowly in fish than in mammals, there is more accumulation of dioxins in seafood. In studies on chicken, beef and pork products have shown that samples of chickens contain more dioxins than other species and are followed by examples of pork and cattle. Furthermore, HxCDD, which is one of the dioxin isomers, was found to be higher in chicken samples (Pohjanvirta and Tuomisto, 1994).

Plant-derived foods are contaminated with PCDD through the application of various pesticides, accumulation of airborne dioxin particles on the plant, and soil. Dioxin usually accumulates in the waxy layer that surrounds the leaf and cannot be easily removed

from the leaf when washed with water. As a result of a study, it was determined that plants and fungi such as potatoes, carrots, and onions absorb and store PCDDs found in soil and water (Lake et al., 2015). In a study on bioaccumulation of dioxin-like PCBs in plants, it was found that PCB accumulation differs between species and these differences vary depending on properties such as leaf surface area, pore density, and chemical composition of the cuticle. Breastfed children may also be exposed to dioxin through breast milk. According to a study conducted in Germany, the amount of PCDD in breast milk was found to be much greater than in cow's milk. As a result of a study conducted in 2006 in EU member states, the minimum and maximum values of the amount of dioxin in some food groups were determined (Trocinio et al., 2012; UNEP, 1999).

#### **1.4. Permitted Legal Levels**

TEQ values the difference between dioxin and similar components according to countries. The daily tolerance value (TDI) set by the Japanese government is 4 pg/kg/day; In Canada and some European countries, the daily tolerance (TDI) is 10 pg/kg/day; WHO (World Health Organization) recommended (TDI) is 2 pg/kg/day; The Minimum Risk Level (MRL) recommended by ATSDR (American Agency for Disease Registry and Toxic Substances) is reported as 1 pg/kg/day. In light of this information, the amount of dioxin and similar components that an average person with 70 kg will receive daily is 70 pg/day given the MRL level. Also, the study conducted in people located and working in different locations also appears to be of interest. The difference between the inhabitants of the town, the inhabitants of the city and the workers of the chemical factory has been made clear. Almost MRL-level dioxin and furan are observed in the blood and breast milk of workers in chemical-producing factories. In addition,



many negatives affecting health are observed in people exposed to dioxin, such as immune systems disorders, neurotoxicity, cardiovascular system disorders, reproductive disorders, developmental disorder, hypertension, and asthma attack as well as birth defects such as chloracne, wasting syndrome, kidney development disorders and especially cancer. (WHO, 1987; WHO, 1997; UNEP, 1999; Hauser, 2002; Schechter et al., 2003; Trocino et al., 2012; Wittsiepe, et al., 2015).

## **2. Conclusions**

Dioxins, which are mostly released as a result of industrial processes and take a long time to be eliminated from nature, cause serious contamination in foods. In terms of human health and food safety, prevention or minimization of dioxin contamination is of paramount importance. Dioxin researches will certainly help us to understand the importance of protecting human health, managing appropriate hazardous substances and minimizing the release of these substances to the environment. As a result, it is very important to prevent dioxin contamination or minimize contact in terms of human health and food safety.



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**SWELLING AND  
RELEASE BEHAVIOR  
OF CARRAGEENAN  
HYDROGELS  
CONTAINING  
GIBBERELIC ACID**

**BÖLÜM**

**2**

**Gülen Oytun AKALIN<sup>1</sup>**

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## Introduction

Gibberellic acid (GA) is known as gibberellin, a pentacyclic diterpene acid supporting growth and elongation of cells. It promotes germinating seeds; it produces mRNA molecules for hydrolytic enzymes. It is usually used in concentrations between 0.01 and 10 mg/L [1,2].

The general purpose of GA can be listed as follows [3,4]:

- Stimulate the growth of stems in genetically stunted plants,
- To facilitate germination of light sensitive seeds,
- Accelerate the growth of leaves,
- Stimulate cell division and cell elongation to ensure growth in the plant stem,
- To extend the durability of ornamental plants,
- To stimulate flowering in some plants,
- Increasing fruit durability and yield,
- Improving fruit size and color,

Misapplications of GA as an example [5].;

- Overdose usage,
- Inaccurate determination of application time,
- Inadequate sensitivity to dose adjustment and application,
- Implementation without considering environmental and health risks.

The controlled release can reduce the negative effects of their misapplications.

This application can provide some advantages such as;

- (1) decreasing gibberellin loss rate,
- (2) supplying nutrients sustainable,
- (3) lowering application frequency and
- (4) minimizing potential negative effects associated with over dosage.

Hydrogels are cross-linked, three-dimensional network polymers capable of swelling without dissolution in water. The cross-linked structure of a hydrogel is given in Figure 1 [6,7]. They are used for many applications due to their excellent properties [8,9]. The controlled delivery systems are one of these applications.

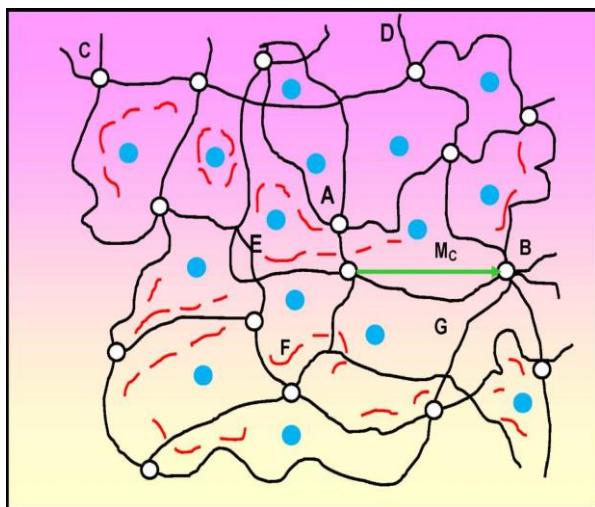


Figure 1. Structure of cross-linked hydrogel. A: Four-functional cross-links, B: Multi-functional cross-links, C-D: Chain points, E: Mixed polymer chains, F: Small cross-linked chains,  $M_c$ : Molecular mass of the chain between two cross-link centers, G: The blank between the cross-

*links, O: Solvent diffused into the blank between the cross-links*

In this work, it was aimed to prepare Carrageenan hydrogels containing GA to determine swelling and release behaviors. The release kinetics parameters and mechanism of gibberellic acid from hydrogel were also determined.

## **Experimental**

### **Materials**

Carrageenan (300.000 g/mol Fluka Chemie), glutaraldehyde (25 wt% in H<sub>2</sub>O) solution, gibberellic acid and phosphate-buffered saline (PBS) tablets were purchased from Sigma-Aldrich.

### **Preparation of Carrageenan hydrogels**

Carrageenan hydrogel was obtained by chemical crosslinking method using glutaraldehyde as a crosslinker. 2 ml of glutaraldehyde solution was mixed with 10 ml of carrageenan aqueous solution (0.5%), the mixture was stirred continuously for 30 min at 80 °C. The mixture was transferred into glass tubes and waited for 24 h at room temperature. Then, the glass tube was broken to obtain cylinder-shaped hydrogel rods. The obtained hydrogels were washed several times with water to remove unreacted chemicals. Then, they were dried in a vacuum oven at 40 °C.

The components used to prepare hydrogels Table 1.

Hydrogel	Carrageenan (%)	Glutaraldehyde (mM)
CRG-1	0.5	0.4
CRG-2	0.5	0.6
CRG-3	0.5	0.9
CRG-4	0.5	1.0

### The loading of GA in Carrageenan hydrogels

The loading of GA was performed during the gelation procedure. Firstly, 10 mL of carrageenan solution (0.5%) was kept under vigorous stirring and the 2 ml of GA solution (0.1 mg/L in ethyl alcohol-water) was put into this solution. After dissolution of the GA, 2 mL of glutaraldehyde solution was added.

#### The water uptake

The water uptake (Q) was specified by being weighed swollen hydrogels in PBS at a certain time. The measurement was applied at room temperature, pH 7.0. The excess surface water was removed by being wiped with paper. Q was given by [10,11];

$$Q = \frac{(m_s - m_d)}{m_d} \quad (1)$$

where  $m_s$  is the mass of the swollen hydrogel and  $m_d$  is the mass of the dry hydrogel.

The temperature test was performed by swelling hydrogels in PBS solution (pH 7.0) at different temperatures. pH test was applied by swelling hydrogels

in PBS solution (at room temperature and different pH values).

### **The morphology analysis**

The morphology analysis was done with a JEOL, JSM 6060 LV SEM. Swollen hydrogels were frozen at  $-80^{\circ}\text{C}$  for 24 h and then taken into Labconco FreeZone 4.5 Freeze Dryer. Hydrogels were coated with gold using a Polaron SC 502 Sputter Coater.

### **The release of GA from hydrogels**

GA was analyzed by spectrophotometer in a Unicam UV-2100 instrument at the wavelength of 305 nm. A certain weighed of dried hydrogels (0.2 g) was taken in 50 mL of ethyl alcohol-water at room temperature, and the sample were taken at certain time intervals in release medium for UV analysis. The solvent samples were poured back into the dissolution cell to keep the volume constant [12];

The cumulative release (%) was measured with the following Equation 2:

$$\text{Cumulative Release}\% = \frac{W_t}{W_{total}} \times 100 \quad (2)$$

where  $W_t$  is the amount of the released drug at any time and  $W_{total}$  is the initial total GA amount in samples.

## **Results and Discussions**

### **The water uptake results**

Figure 2 displays the water uptake for hydrogels with time. Q firstly increased with time and then remained

stable at 12 h for all hydrogels. CGR-4 hydrogel exhibited a higher sorption rate than others. The results were in good agreement with the literature. Many reports have been explained that crosslinker density and crosslinker type affects  $Q$  value [13-15].

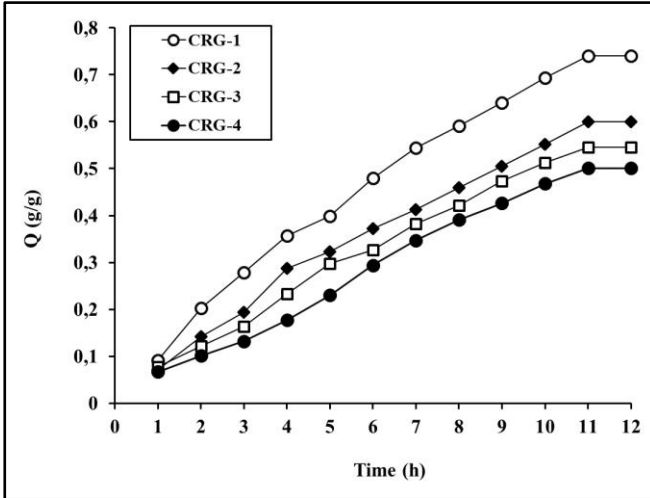


Figure 2. The water uptake of hydrogels with time



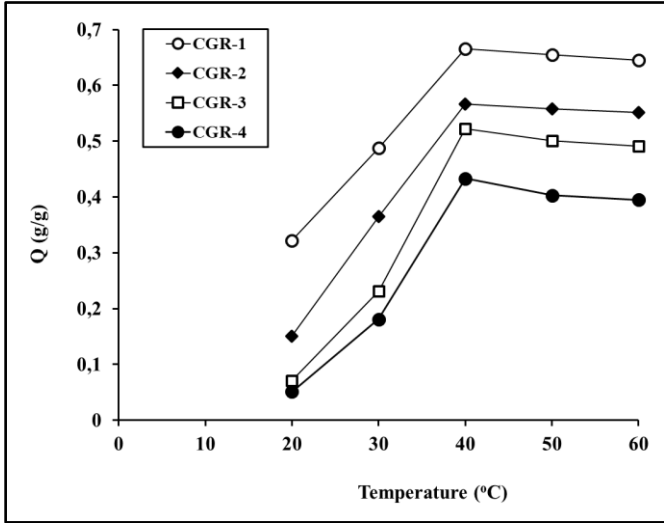


Figure 3. The water uptake of hydrogels with temperature

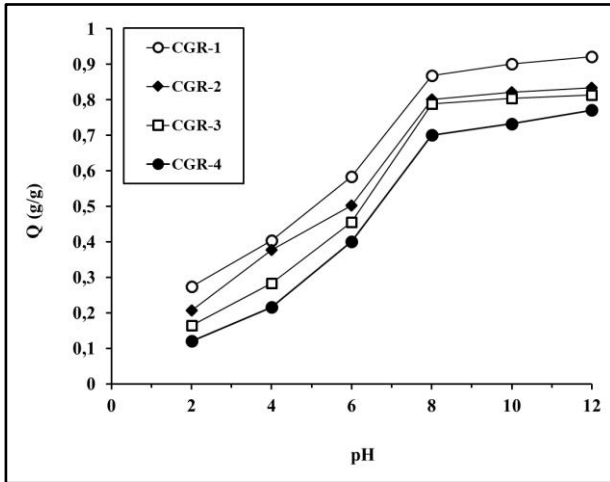


Figure 4. The water uptake of hydrogels with pH

Figure 3 displays that  $Q$  values increased with temperature at the beginning. This can be explained by the increase the thermal mobility of the molecules in

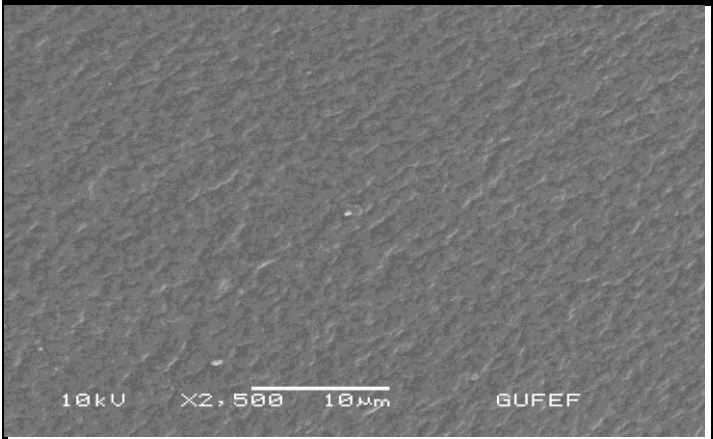
structure. Above 40°C, Q values were not changed so much. Figure 4 displays that all hydrogels exhibited higher Q values in basic medium in comparison with acidic medium. This can be explained by increasing the ionic properties of the structure with pH. In the acidic environment, protonation of the ionic groups can be occurred, so the swelling is precluded. In basic medium, the electrostatic repulsion between ionic groups can enhanced due to ionization in the hydrogel structure, so the easy swelling occurs [14,15].

### **SEM analysis**

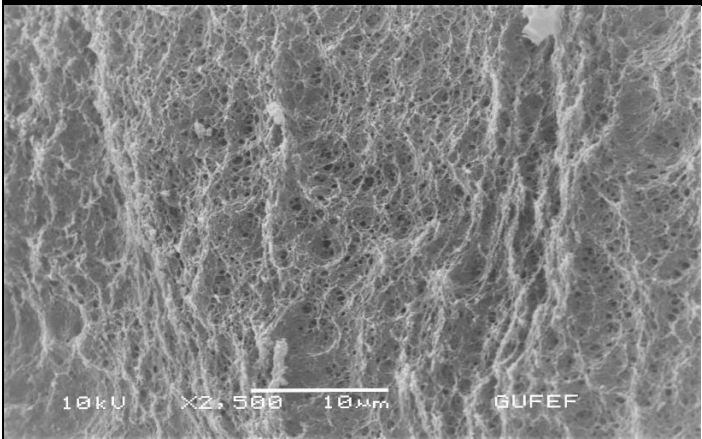
The SEM observations of samples are showed in Figure 5. The morphology of dry hydrogel is too different with that of swollen hydrogel. Dry sample had nonporous structure, while swollen hydrogel existed small pore structure. This porosity leads to easy penetration of water into hydrogel structure.

### **The release of GA from hydrogels**

The cumulative release results are given in Figure 6. The releasing increased with time and the equilibrium values was reached at nearly 20 h. The releasing values are in good agreement with the water uptake test. The release values decreased with increasing glutaraldehyde concentration. As emphasized before, the highest water uptake result was obtained for CRG-1 hydrogel because of lower crosslinker concentration. While hydrogel swelled easily, the loaded GA molecules easily diffused through the pores of hydrogel structure [16-18].



(a)



(b)

Figure 5. The SEM images of (a) dry hydrogel, (b) swollen hydrogel

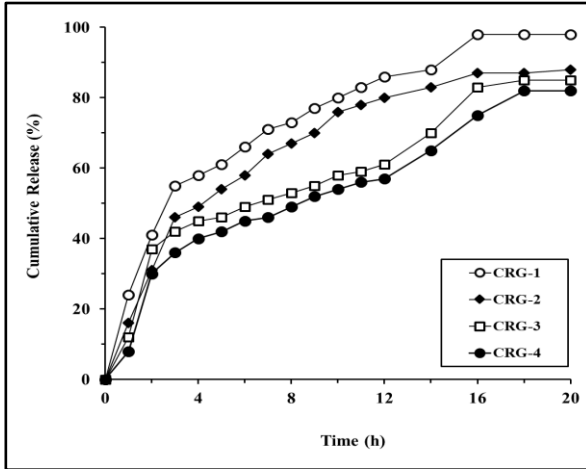


Figure 6. The cumulative release of GA from hydrogels

## Conclusions

Carrageenan hydrogels were prepared by chemical crosslinking for the controlled release of GA. The water uptake (Q) tests were carried out with time temperature, pH. The Q values were decreased with increasing concentration of GA. The SEM analysis was done for dry and swollen hydrogels. GA release from hydrogels were also performed. The releasing values decreased with glutaraldehyde concentrations.

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**PREPARATION OF  
MELATONIN LOADED  
SODIUM  
ALGINATE/GELATIN  
HYDROGELS**

**BÖLÜM  
3**

**Gülen Oytun  
AKALIN<sup>1</sup>**

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## Introduction

Melatonin (N-acetyl-5-methoxytryptamine) is an organic compound with a melting point of 116-118 ° C. Melatonin is secreted from the cells of the pineal gland called "pineolytic". Melatonin determines or affects biorhythmia. Melatonin helps to increase sleep in the brain at night. Melatonin is also an antioxidant that acts with free radicals and is believed to be able to prevent cancer. The chemical structure of melatonin is given in Figure 1 [1,2].

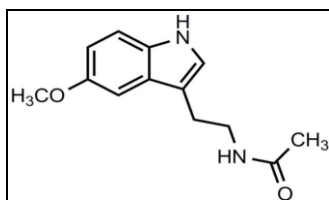


Figure 1. Chemical structure of melatonin

Alginate is a biocompatible, mucosal and biodegradable polysaccharide polymer. It is a copolymer of consecutive  $\beta$ -D-mannuronic acid (M bloks) and  $\alpha$ -L-gluronic acid (G bloks) monomers. The length and distribution of the M and G bloks along the polymer chain varies depending on the alginate source. Alginate is abundant in nature as a structural component of brown algae in the sea. The alginate is a hydrophilic polymer and takes the form of hydrogel in water. The chemical structure of sodium alginate is given in Figure 2. Due to their polyacidic properties, it undergoes reversible gelation in aqueous solutions. The internal chain ionic bridges are occurred by the interaction between divalent cations (Ca<sup>2+</sup>, Zn<sup>2+</sup> etc.) and the G units of alginate [3-5].

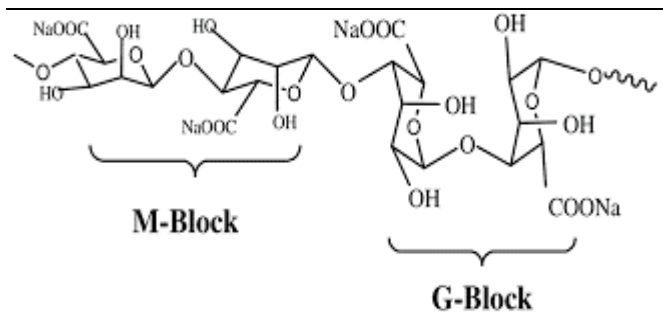


Figure 2. Chemical structure of sodium alginate

Gelatin is a pure protein obtained by hydrolysis of collagen found in animal skin and/or bones. The chemical structure of gelatin is shown in Figure 3. It contains 85-90% protein, 10-12% water and 2-3% minerals. Gelatin is widely used in various fields of the food industry due to its properties such as thickening, gelation, clarification, stabilization, foaming, emulsion formation and stabilization, film formation [6,7].

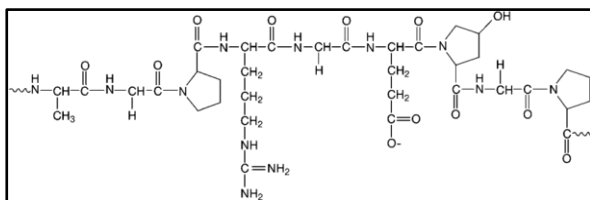


Figure 3. Chemical structure of gelatin

The aim of this work was to produce polymeric material for controlled releasing of melatonin. The material was synthesized as hydrogel using two natural polymers. Inner part was produced from sodium alginate hydrogel and this core was surrounded by gelatin shell. The water uptake property of synthesized hydrogel was investigated, and the melatonin release experiment was also examined.

## Experimental

### Materials

Sodium alginate, gelatin, melatonin and phosphate-buffered saline (PBS) tablets were purchased from Sigma-Aldrich. Calcium chloride, Tween 80 were obtained from Merck.

### Preparation of Carrageenan hydrogels

Firstly, petri dishes were filled with Vaseline. Petri dishes kept waiting at  $-18\text{ }^{\circ}\text{C}$  for 1 h. Then, several cavities were formed on the surface. Sodium alginate solution (10%) was put into the cavities, petri dish was left at  $-18\text{ }^{\circ}\text{C}$  for 45 min. The solidified sodium alginate beads were taken in 100 mL of  $\text{CaCl}_2$  (5%) solution for 15 min, 30 min and 45 min. The collected beads were washed several times with distilled water and dried first in air and then in a vacuum oven at  $40^{\circ}\text{C}$ . The sodium alginate beads were coated by immersing them into gelatin solution (%10). The coated beads were dried at  $40\text{ }^{\circ}\text{C}$  in vacuum and they were stored for further use. The conditions were presented in Table 1.

*The components used to prepare hydrogels Table 1.*

Hydrogel	Sodium Alginate (%)	Gelatin (%)	$\text{CaCl}_2$ (%)	Reaction Time (min)
Alg-Gl-1	10	10.0	5	15
Alg-Gl-2	10	10.0	5	30
Alg-Gl-3	10	10.0	5	45

### The water uptake

The water uptake (Q) was determined by being weighed swollen hydrogels in PBS at regular intervals. The experiment was performed at room temperature and pH 1.5 or 7.4. The excess surface water was removed by

being wiped with filter paper. Q was calculated by using Equation 1 [8,9].

$$Q = \frac{(m_s - m_d)}{m_d} \quad (1)$$

where  $m_s$  is the weight of the swollen hydrogel and  $m_d$  is the weight of the dry hydrogel.

### **The preparation of melatonin loaded sodium alginate/gelatin hydrogels**

The loading of melatonin was applied during the gelation procedure [10,11]. 10 ml of sodium alginate solution (10%) and 1.0 mL of melatonin solution (1.0 mg / mL in ethanol) were kept under vigorous stirring for 1 h. Then, the mixture solution was put into the cavities, petri dish was left at -18 °C for 45 min. The solidified beads were taken in 100 mL of CaCl<sub>2</sub> (5%) solution for 15 min, 30 min and 45 min. The collected beads were washed several times with distilled water and dried first in air and then in a vacuum oven at 40°C. The beads were coated by immersing them into gelatin solution (%10). The coated beads were dried at 40 °C in vacuum. The melatonin loaded sodium alginate/gelatin hydrogels were stored for further use.

### **The release of melatonin from hydrogels**

Melatonin was analyzed by spectrophotometer in a Unicam UV-2100 instrument at the wavelength of 278 nm [12]. The release tests were performed in pH 1.5 and pH 7.4.

In order to investigate the releasing of melatonin from hydrogels, pH 1.5 medium was prepared as follows: 1.65 mL of 37% HCl to a flask was completed to 100 mL with distilled water. 1.4911 grams of KCl were weighed and

dissolved in 100 ml of distilled water. 82.8 mL of the prepared HCl solution (0.2 M) was added to 100 mL KCl solution (0.2 M) and mixed. 100 mL of this solution was taken into another beaker and the pH was adjusted to 1.5 [13].

In order to investigate the releasing of melatonin from hydrogels, pH 7.4 medium was prepared as follows: A tablet of PBS was taken into 90 mL of distilled water and the solution was stirred. When the tablet was completely dissolved, the pH of the solution was measured with the pH meter. When the pH of the solution was less than 7.4, the pH was adjusted to 7.4 by dropwise addition of 0.5 M NaOH solution. 1% Tween 80 solution was prepared, and 1.5 mL of this solution was added to 50 mL pH 7.4 medium [13].

The dried hydrogel was immersed into 50 mL release medium. 2.0 mL of sample was taken from the release medium every 30 minutes and centrifuged at 18000 rpm and 4°C for 30 minutes. The absorbance of the supernatant of the centrifuged sample was measured against the pH 1.5 at 278 nm on the spectrophotometer. This procedure was repeated for 8 hours.

The cumulative release (%) was measured with the following Equation 2 [14,15] :

$$\text{Cumulative Release\%} = \frac{W_t}{W_{total}} \times 100 \quad (2)$$

where  $W_t$  is the amount of the released drug at any time and  $W_{total}$  is the initial total drug amount in samples.

## Results and Discussions

### The water uptake results

Figure 4 shows the water uptake for hydrogels with time. Q values increased with time for all hydrogels and then the equilibrium values were reached near 8 h. The crosslinking reaction time affected Q values. As the crosslinking reaction time was increased from 15 min to 45min, Q value was decreased. Alg-GI-1 hydrogel displayed a higher swelling rate than others.

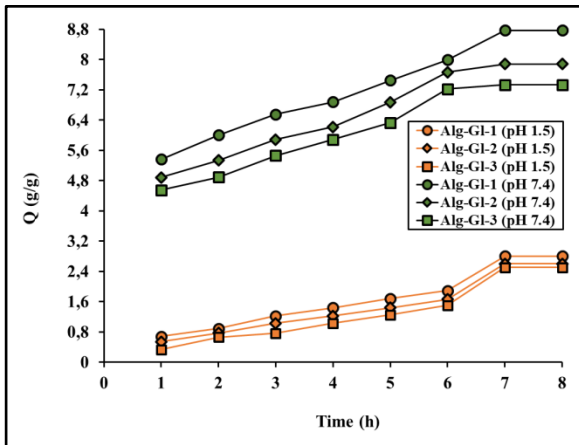


Figure 4. The water uptake of hydrogels with time

The sodium alginate/gelatin hydrogels exhibited much more water uptake behavior in small intestine (pH 7.4) than in gastric (pH 1.5). This result shows that the swelling properties of hydrogels affect the change of pH. The increment of Q value can be explained by the exchange of ions between calcium ions in the hydrogel structure and sodium ions in the phosphate buffer [16]. Alginate polymer has a very hydrophilic structure due to  $-OH$  and  $-COOH$  groups in its chain. At neutral pH,



water diffuses easily into structure to form hydrogen bridges, so hydrogel gains volume and swell [17].

### The release of melatonin from hydrogels

The absorbances of melatonin solutions at different concentration range to be studied at 278 nm were determined due to follow the release experiments. The absorbances versus concentrations were plotted. The graph is given as Figure 5.

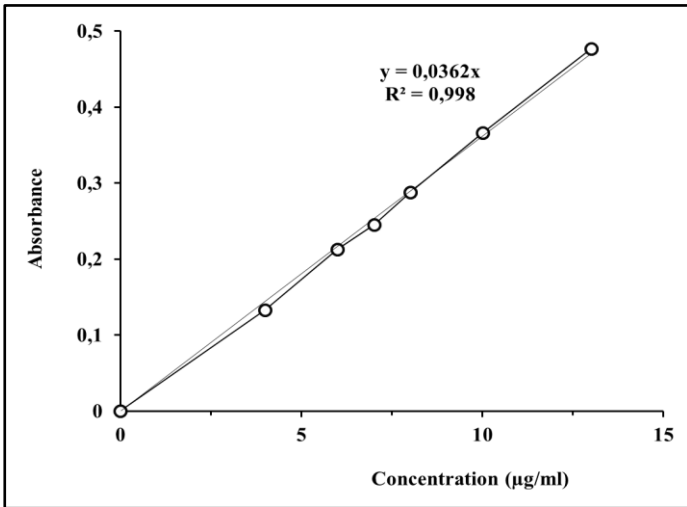


Figure 5. The working graph of melatonin

The cumulative release results at pH 1.5 are given in Figure 3. The values increased with time and the equilibrium values was observed at nearly 8 h. The releasing values are in concordance with the water uptake results. The release values decreased with increasing the crosslinking reaction time. The higher reaction time decelerates diffusing of the water through the pores of hydrogel structure [17,18].

The cumulative release results at pH 7.4 are given in Figure 4. The melatonin loaded sodium alginate/gelatin

hydrogels exhibited much more releasing behavior in small intestine (pH 7.4) than in gastric (pH 1.5). The results are in good agreement with water uptake results. At pH 7.4, the increasing the exchange of ions between calcium ions in the hydrogel structure and sodium ions in the phosphate buffer enhances the melatonin release rate.

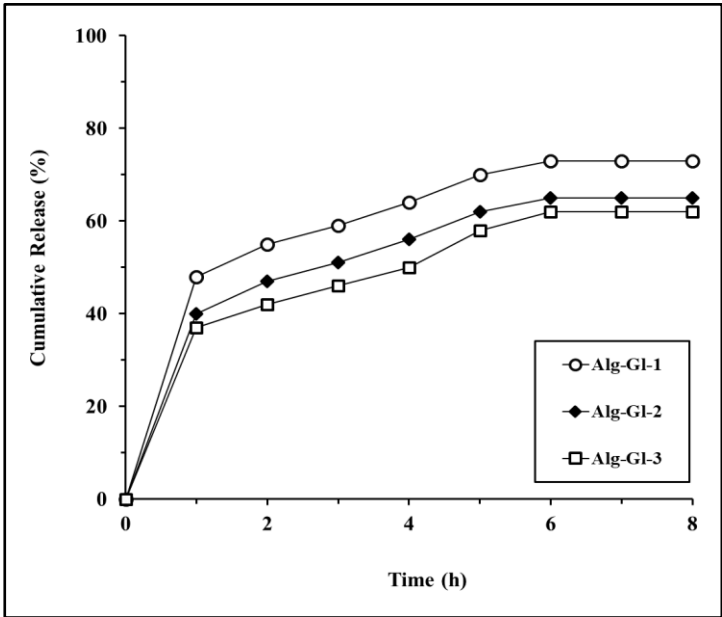


Figure 6. The cumulative release of melatonin at pH 1.5

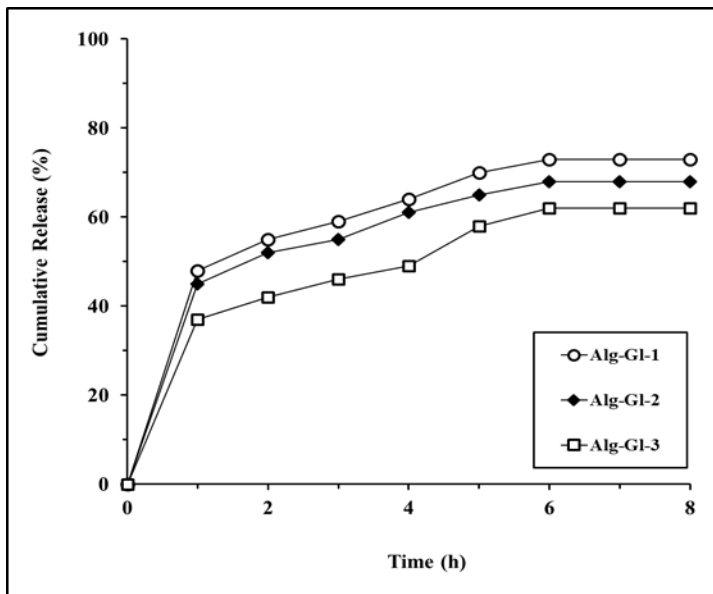


Figure 7. The cumulative release of melatonin at pH 7.4

## Conclusions

In this study, sodium alginate / gelatin hydrogels with different crosslinking reaction times were prepared. The water uptake (Q) test of hydrogels was performed in vitro gastric (pH 1.5) and small intestine (pH 7.4) environment. The higher Q values were observed in pH 7.4. This could be explained by interaction between calcium ions of alginate and sodium ions of phosphate buffer. The release behavior of melatonin loaded sodium alginate/gelatin hydrogels were also investigated. Release studies in vitro pH 1.5 and pH 7.4 gave significantly different results. Release studies were in good accordance with water uptake tests.



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**MEASUREMENT OF  
THE ELASTIC  
PROPERTIES OF  
ORTHOTROPIC  
MATERIALS BY  
ULTRASONIC  
METHOD\***

**BÖLÜM  
4**

**Hatice GÜZEL<sup>1</sup>,  
İmran ORAL<sup>2</sup>,  
Hüseyin İŞLER<sup>3</sup>**

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\* This chapter of the book is produced from the master of science thesis titled: The ultrasonic measurement of elasticity coefficient of fiber reinforced composite materials.

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## 1. INTRODUCTION

Materials science has become increasingly important in 21<sup>th</sup> century with the advancement of science and technology and the need for materials with different characteristics. As material science is developed with the developing technology, composite materials can be produced lighter, they are produced in a way that they are more resistant to external conditions and their costs have been reduced (Erkan, 2009). In our daily life, the applications of composite materials has increased so much that it can be seen at everywhere around us. Composite materials are generally used in areas such as sports equipment, prosthesis body organs such as arm and leg, airplanes, helicopter engines, rocket, missile body and automotive area (Demircioğlu, 2006). It is very important to know the mechanical properties of the composite materials. It is ot suitable to use these materials without determination of their all the characteristic and mechanical properties of the materials(Sakin, 1994).

It is vital to identify the properties of composite materials produced with new properties and to perform quality checks of these materials before they are used. Because it is necessary to know which properties are better or weaker in terms of which purpose a new material is produced. In addition, they can take various damages both during production and after transportation and during assembly. Therefore, many destructive methods (tensile test, compression test, torsion test, etc.) and non-destructive method are used in order to reveal the different properties of the materials produced. Recently, the use of ultrasonic methods in non-destructive material testing has increased considerably and has become preferable to damaged methods.

The basic operating principle of the ultrasonic testing method is based on the damping, scattering and reflection

of high-frequency ultrasonic waves as a result of interaction with media particles as they pass through the material (Abi, 2007). In the ultrasonic test, many elasticity parameters can be measured without any damage by the help of measured longitudinal and shear ultrasonic wave velocities in the materials and by known density values of the materials.

In this study, elasticity constants, Young modulus, Poisson ratios and Shear modulus of orthotropic E-glass/epoxy and carbon/epoxy composites made of a special company (Izoreel, Turkey) were calculated by using ultrasonic testing. In the research, the effect of E-glass and carbon fibers, which are arranged in different directions, on the material properties were examined by ultrasonic method and the results obtained were examined.

## **2. MATERIALS AND METHOD**

### **2.1 Synthesis of materials**

In the study; E-glass fibers (Sisecam-Turkey) reinforced epoxy composite plates (500mmx500mmx12,5mm) and carbon fiber (Aksa-Turkey) reinforced epoxy composite plates (500mmx500mmx12,5mm) supplied from Izoreel Composites-Turkey. 50 layers of fiber are used for E-glass/epoxy composite and 49 layers of fibers are used for carbon/epoxy composite plates. Epoxy resin produced by using two artificial resins (100 units from Bakelite EPR840, 80 parts from Bakelite EPH875) used as matrix material in both materials. The general characteristics of the epoxy resin and fibers used in the study are given in Table-1 below.

**Table-1.** Properties of epoxy resin and fibers

<b>Mechanical properties</b>	<b>Epoxy resin</b>	<b>E-glass fiber</b>	<b>Carbon fiber</b>
Tensile strength (MPa)	70-80	2400	4200
Young's modulus (MPa)	3,5	73	240

In the process of producing composite materials, firstly EPR840 and EPH875 epoxy resins were mixed in 100/80 ratio and heated up to 70 0C to obtain the matrix material. This matrix system is applied to the E-glass and carbon fibers by roll and stacked on top of each other. 50 layers of fiber used for e-glass while 49 layers of fiber used for carbon. The composites have thickness of 12,5 mm obtained and they were hardened at 120 0C for 4 hours.

## **2.3 Experimental Measurements**

### **2.3.1 Density measurements of materials**

The density measurements of the two types of composite materials used in the study were performed at room temperature using Radwag analytical balance (Radwag AS220/C/2, Poland) and Density kit (Radwag 220, Poland). Densities were measured according to the Archimedes' principle, and distilled water was used as immersion liquid. For density measurement, a small piece of E-glass/epoxy and carbon/epoxy composites without air gap was prepared. The room temperature at the time of the density measurement is read from the thermometer on the density kit and entered into the analytical balance. After measuring the mass of materials in the air and in the water by the density kit built on the radwag analytical balance, the density values of the materials were determined automatically with Radwag AS220/C/2 analytical balance with 0.01% accuracy.

### 2.3.2 Ultrasonic velocity measurements

The 5800PR ultrasonic pulse generator-receiver device (Panametrics Olympus NDT, USA) and the GW Instek GDS-2062 model 60 MHz digital oscilloscope (Taiwan) were used. Two pieces of 2,25 MHz shear wave transducers (V154-Panametrics Olympus NDT, USA) for measuring ultrasonic shear wave velocities in both E-glass fiber reinforced epoxy composite and carbon fiber reinforced epoxy composites and two pieces of 5 MHz longitudinal wave transducers (V109- Panametrics Olympus NDT, USA) were used. The through transmission method was used in the velocity measurements of ultrasonic shear waves, while the velocity measurements of the longitudinal ultrasonic waves were performed by pulse-echo method,. For this reason, the thicknesses of the materials and the transition times of the ultrasound waves were determined by the following equation-1.

$$V = \frac{2d}{\Delta t} \quad (1)$$

Where  $d$  and  $\Delta t$  in Equation (1) is the material thickness and the time passed through materials, respectively. On the other hand, ultrasonic shear wave velocities were determined using  $d$  instead of  $2d$  in equation-1 due to the use of through-transmission method. These velocity measurements were made by direct contact technique. In the direct contact technique, a coupling liquide is used to facilitate the transmission of sound between the material examined and the ultrasonic transducer and to minimize the reflections. Because the acoustic impedance of the air is very low compared to the acoustic impedance of solid materials. This impedance difference cause the reflection of the sound waves from the surface of the examined sample and decrease the sound energy passing through the medium. In this study, SWC (Olympus NDT, USA) coupling liquid is used for

shear wave velocity measurements and BQ (Olympus NDT, USA) coupling liquid is used for longitudinal wave velocity measurements as well.

### **2.3.3 Optical microscope and combustion experiments**

It is very important to reveal the structure of the materials used in ultrasonic research. Because there are two independent elasticity coefficients in isotropic materials which their physical properties do not change depending on orientation (Balci, 2011), while this number can be up to 21 in non-isotropic materials which especially have not got any symmetry in any direction (Örçen, 2005). For this reason, the material surfaces of both composites were sanded thoroughly with fine sandpaper and then they were first taken with optical microscope (Olympus, USA) and then subjected to combustion test.

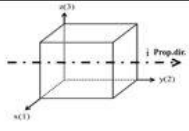
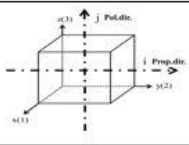
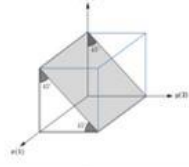
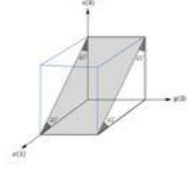
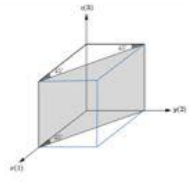
The combustion test is a technique which remove the matrix by means of a solvent capable of dissolving the matrix in fiber-reinforced composites without affecting the fiber. The easiest way to remove the matrix in such composites is to burn the epoxy resin in the oven at around 600 °C. Burning of the matrix at such temperatures which are very low from the melting points of carbon or glass fibers does not cause any physical changes in glass or carbon fibers (Akdemir, 1992). The combustion test is usually carried out to determine the fiber-to-resin ratio of fiber-reinforced composites. However, the combustion test was made to determine the fiber orientation of the materials in this study. Therefore, in order to determine fiber orientations in E-glass/epoxy and carbon/epoxy composite materials used in this study, these materials were burned in the oven at 600 °C until the epoxy matrix was completely evaporated.

### **2.3.4 Calculation of Elasticity Coefficients**

As a result of the optical microscope and combustion experiments, it was determined that the materials examined had an orthotropic structure. The orthotropic material is the material with three elastic symmetry planes perpendicular to each other (Küçüksucu, 2011). Such materials have 9 independent elasticity constants (Tatar, 2011). The elasticity constants in the orthotropic material are  $C_{11}$ ,  $C_{22}$ ,  $C_{33}$ ,  $C_{44}$ ,  $C_{55}$ ,  $C_{66}$ ,  $C_{12}$ ,  $C_{13}$  and  $C_{23}$ . In order to calculate these coefficients, the density values of these materials and the longitudinal and shear ultrasonic wave velocities in these materials should be measured in different directions. For the calculation of these elasticity coefficients, four samples of these materials are needed. The required sample types (A, B, C and D) and the ultrasonic velocity values to be measured are given in Table 2 below.



**Table-2.** Types of materials used in velocity measurements  
(Prop. dir. = Propagation direction, Pol. dir. = Polarization direction)

Ultrasonic Speeds	Sample type / Measured speed type	Material shape
$V_{ii} \begin{cases} V_{11} \\ V_{22} \\ V_{33} \end{cases}$	A / Longitudinal wave velocity	
$V_{ij} \begin{cases} V_{12} \\ V_{13} \\ V_{23} \end{cases}$	A / Transverse wave velocity	
$V_{s1}$	B / Transverse wave velocity	
$V_{s2}$	C / Transverse wave velocity	
$V_{s3}$	D / Transverse wave velocity	

Sample A was used to measure shear wave velocity values ( $V_{ij}$ ) and longitudinal waves values ( $V_{ii}$ ) in x, y and z directions. Sample B is the sample obtained by cutting the material to the angle of  $45^{\circ}$  with the y and z axes. Sample C is the sample obtained by cutting the material to an angle of  $45^{\circ}$  with x and z axes. Sample D is

the sample obtained by cutting the material to the angle of  $45^{\circ}$  with x and y axes.

The vibration direction of the particles in the material and the direction of propagation of the ultrasonic waves used in the calculation of elastic coefficients in orthotropic materials are given in Table 3.

**Table-3.** *Ultrasonic wave velocities propagated in orthotropic materials and their propagation directions*

Ultrasonic wave velocity (m / s)	Wave type and propagation direction	Vibration direction of particles
$V_{11}$	Longitudinal wave propagating along x-axis.	x-axis
$V_{22}$	Longitudinal wave propagating along y-axis	y-axis
$V_{33}$	Longitudinal wave propagating along z-axis	z- axis
$V_{12}$	Shear wave propagating along x-axis	y-axis
$V_{13}$	Shear wave propagating along x-axis	z- axis
$V_{23}$	Shear wave propagating along y-axis	z- axis
$V_{s1}$	Shear wave propagating in sample B	Perpendicular to x- axis
$V_{s2}$	Shear wave propagating in sample C	Perpendicular to y-axis
$V_{s3}$	Shear wave propagating in sample D	Perpendicular to z-axis

The relations between the densities of orthotropic materials ( $\rho$ ), elasticity constants, ultrasonic wave velocities and mechanical properties are determined by plane waves emitted in the appropriate directions in these materials and are expressed as follows (Mistou et al., 1999):

$$\begin{aligned}
 C_{11} &= \rho \cdot V_{11}^2, & C_{44} &= \rho \cdot V_{23}^2 \\
 C_{22} &= \rho \cdot V_{22}^2, & C_{55} &= \rho \cdot V_{13}^2 \\
 C_{33} &= \rho \cdot V_{33}^2, & C_{66} &= \rho \cdot V_{12}^2
 \end{aligned}$$

$$\begin{aligned}
 C_{23} &= \\
 &= \sqrt{(C_{22} + C_{44} - 2\rho \cdot V_{s1}^2) \cdot (C_{33} + C_{44} - 2\rho \cdot V_{s1}^2) - C_{44}} \\
 &\quad (2)
 \end{aligned}$$

$$\begin{aligned}
 C_{13} &= \sqrt{(C_{11} + C_{55} - 2\rho \cdot V_{s2}^2) \cdot (C_{33} + C_{55} - 2\rho \cdot V_{s2}^2)} \\
 &\quad - C_{55} \\
 C_{12} &= \sqrt{(C_{11} + C_{66} - 2\rho \cdot V_{s3}^2) \cdot (C_{22} + C_{66} - 2\rho \cdot V_{s3}^2)} \\
 &\quad - C_{66}
 \end{aligned}$$

Young modules ( $E_1, E_2, E_3$ ), Poisson ratios ( $\nu_{13}, \nu_{12}, \nu_{23}$ ) and Shear modulus ( $G_{12}, G_{13}, G_{23}$ ) of the materials used in the study were calculated using the equations given in equation (2) and equation (3) below (Mistou et al., 1999):

$$D = C_{11} \cdot C_{22} \cdot C_{33} - C_{11} \cdot C_{23}^2 - C_{22} \cdot C_{13}^2 - C_{33} \cdot C_{12}^2 + 2 \cdot C_{12} \cdot C_{13} \cdot C_{23}$$

$$E_1 = \frac{D}{(C_{22} \cdot C_{33} - C_{23}^2)}, \quad E_2 = \frac{D}{(C_{11} \cdot C_{33} - C_{13}^2)}, \quad E_3 = \frac{D}{(C_{11} \cdot C_{22} - C_{12}^2)}$$

$$G_{23} = C_{44}, \quad G_{13} = C_{55}, \quad G_{12} = C_{66} \quad (3)$$

$$v_{23} = \frac{-E_2(C_{12} \cdot C_{13} - C_{23} \cdot C_{11})}{D},$$

$$v_{13} = \frac{-E_1(C_{12} \cdot C_{23} - C_{13} \cdot C_{22})}{D}$$

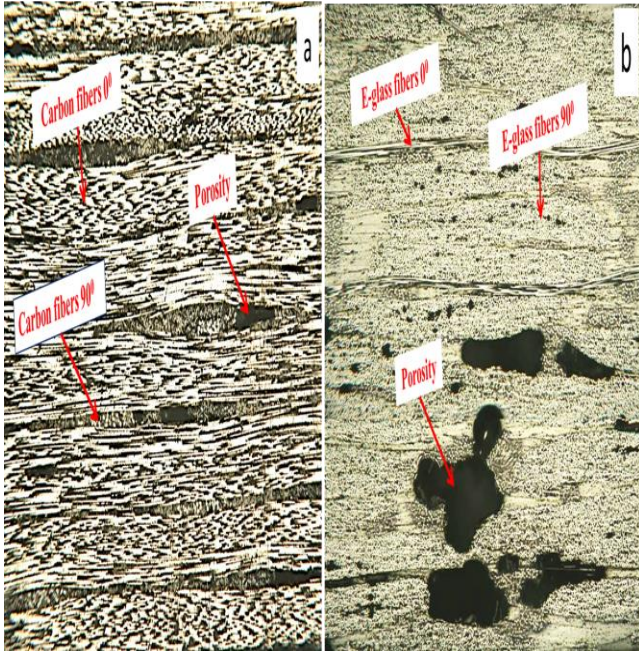
$$v_{12} = \frac{-E_1(C_{13} \cdot C_{23} - C_{12} \cdot C_{33})}{D}$$

### 3. FINDINGS

In this chapter; The optical microscope images of the materials examined, the data obtained from the combustion tests, the longitudinal and shear ultrasonic wave velocities measured in all directions are given Fig.1 and Tables 4-10.

#### 3.1 Findings of Optical Microscope and Combustion Tests

The optical microscope images obtained to reveal the arrangement of E-glass/epoxy and carbon/epoxy composites fibers are given in Fig.1.



**Fig.1.** Optical microscope images of E-glass/epoxy and carbon/epoxy composites

As seen from Figure-1, fiber orientation of both e-glass/epoxy and carbon/epoxy composites is  $0^0/90^0$ . Therefore, after the epoxy matrix is applied on the fibers arranged at  $0^0$  degrees, it is seen that they are produced by repeating a new fiber sequence in  $90^0$ . Figure-1 also shows that there are porosities in both composites. This causes a decrease in the mechanical properties of these materials in the direction of porous structures.

The combustion test is usually carried out to determine the fiber-to-resin ratio of fiber-reinforced composites. However, this study was conducted to examine the fiber orientation of composites. E-glass/epoxy and carbon/epoxy composites were both burned in the oven at  $600^0\text{C}$  until the matrix was completely evaporated. When the remaining fibers were

collected from the top with tweezers at the end of this process, it was seen that the fibers in both composite materials are in y (2) of  $90^0$  and x (1) of  $0^0$  directions. However, there are no fibers in the z (3) direction, which is the third dimension in both composite materials.

Therefore, it was understood that both materials were orthotropic materials as a result of the burning test on the composites and optical microscope analysis. For this reason, the samples which are given Table 2 were prepared to calculate the elastic properties of these materials by ultrasonic method.

The densities, longitudinal and shear ultrasonic wave velocities of these orthotropic materials were measured to measure their elasticity coefficients by ultrasonic methods. By using these densities and velocities, nine elasticity constants, three Young modulus ( $E_1, E_2, E_3$ ); three Poisson's ratios ( $\nu_{13}, \nu_{12}, \nu_{23}$ ) and three shear modulus ( $G_{12}, G_{13}, G_{23}$ ) of these orthotropic materials were calculated. The data obtained are given in Table 4-10.

### 3.2. Density values of materials

The density, longitudinal and shear wave velocity values of e-glass/epoxy and carbon/epoxy composites are given in Table 4 below.

**Table 4.** Density values of materials

<b>Material type</b>	<b><math>\rho</math> (kg/m<sup>3</sup>)</b>
Epoxy matrix	1100
E-glass / Epoxy	1803
Carbon / epoxy	1506

As it can be seen from Table4, the density of epoxy matrix has increased from 1100 (kg/m<sup>3</sup>) to 1803 (kg/m<sup>3</sup>) by E-glass fiber addition and when the carbon fiber was added to epoxy matrix, it has reached 1506 (kg / m<sup>3</sup>).

### 3.3. Ultrasonic wave velocity values

The longitudinal ultrasonic wave velocities( $V_{11}$ ,  $V_{22}$ ,  $V_{33}$ ) and shear wave velocities ( $V_{s1}$ ,  $V_{s2}$ ,  $V_{s3}$ ) which are measured in the x (1), y (2) and z (3) directions and shear wave velocities ( $V_{s1}$ ,  $V_{s2}$ ,  $V_{s3}$ ) which measured in samples B, C and Dare given in Table 5.

*Table-5. Ultrasonic wave velocities measured for different surfaces of e-glass/epoxy and carbon/epoxy composite materials*

Velocity type (m/s)	E-glass/epoxy	Carbon/epoxy
$V_{11}$	4520	8674
$V_{22}$	3098	2898
$V_{33}$	2165	2490
$V_{12}$	1812	1694
$V_{13}$	1460	1662
$V_{23}$	1272	1299
$V_{s1}$	1426	1411
$V_{s2}$	1648	2047
$V_{s3}$	2253	2183

According to data in Table 5, the longitudinal wave velocity of thee-glass/epoxy composite material is found as 4520 m/s inthe direction of x (1), as 3098 m/s in the direction of y (2) and as 2165 m/s in the direction of z (3). Also,the longitudinal wave velocity of the carbon/epoxy composite material is found as 8674 m/s in the direction of x (1), as 2898 m/s in the direction of y (2) and as 2490 m/s in the direction of z (3). According to these values, the highest ultrasonic wave velocities for both composites were measured in the direction of x (1) while the lowest was found in z (3) as well.

It can be seen from Table5 thatthe velocity of shear wave propagated in the direction of x (1) is measured as 1812 m/s when it polarized in the direction of y (2) and as 1460 m/s when it polarized in the direction of z (3) for e-glass/epoxy composite material. The velocity of the shear wave propagated in the direction of y(2) is measured as 1272 m/s when it polarized in the direction of z (3) for the e-glass/epoxy composite material.

As it can be seen from Table5, the velocity of the shear wave propagated in the direction of x (1) is measured as 1694 m/s when it polarized in the direction of y (2) and as 1662 m/s when it polarized in the direction of z (3) for carbon/epoxy composite material. The velocity of the shear wave propagated in the direction of y(2) is measured as 1299 m/s when it polarized in the direction of z (3) for the carbon/epoxy composite material.

As can be seen in both composite materials, the highest ultrasonic shear wave velocity is measured for the waves propagated in the direction of x (1) and polarized in the direction y (2), while the lowest is measured for the waves propagated in the direction of y (2) and polarized in the direction of z (3).

According to Table 5, shear ultrasonic wave velocities in e-glass/epoxy composite sample  $V_{s1}$ ,  $V_{s2}$  and  $V_{s3}$  are



found as 1426 m/s, 1648 m/s and 2253 m/s, respectively. Also, shear ultrasonic wave velocities in carbon/epoxy composite sample  $V_{s1}$ ,  $V_{s2}$  and  $V_{s3}$  are found as 1411 m/s, 2047 m/s and 2183 m/s, respectively.

### 3.4. Elasticity coefficients of materials

The elasticity constants calculated for both composites in some specific directions by replacing the measured ultrasonic wave velocities and densities of both materials into equation-2 and are given in Table 6.

**Table 6.** *Calculated elasticity constant values for e-glass / epoxy and carbon/epoxy composites*

Coefficient of elasticity (GPa)	E-glass / epoxy	Carbon / epoxy
$C_{11}$	36,8	113,3
$C_{22}$	17,3	12,7
$C_{33}$	8,5	9,3
$C_{44}$	2,9	2,5
$C_{55}$	3,9	4,2
$C_{66}$	3,9	4,3
$C_{12}$	5,1	12,1
$C_{13}$	4,9	5,5
$C_{23}$	4,3	4,8

For calculating the Young's modulus of the materials, the parameter D in equation (3) is calculated for each of the two materials and are given in Table 7.

**Table 7.** *D parameter values calculated for e-glass / epoxy and carbon / epoxy composites*

<b>Composite type</b>	<b>D Parameter (N/m<sup>2</sup>)<sup>3</sup></b>
E-glass / Epoxy	4,28.10 <sup>30</sup>
Carbon / epoxy	9,65.10 <sup>30</sup>

Young's modulus in the x (1), y (2), z (3) directions are calculated by substitution of the elasticity coefficients calculated by equation (2) and the D parameter into the equation (3) and the obtained values are shown in Table 8.

**Table 8.** *Young Modulus (E<sub>1</sub>, E<sub>2</sub> and E<sub>3</sub>) values calculated in the x (1), y (2) and z (3) directions for e-glass/epoxy and carbon/epoxy composites*

<b>Composite type</b>		<b>E<sub>1</sub> (GPa)</b>	<b>E<sub>2</sub> (GPa)</b>	<b>E<sub>3</sub> (GPa)</b>
E-glass / Epoxy	/	33,51	14,94	7,00
Carbon / epoxy	/	101,59	9,38	7,50

As seen from Table 8, the Young's modulus of e-glass/epoxy composite was found to be 33.51 GPa, 14.94 GPa and 7.00 GPa in the directions of x (1), y (2) and z (3), respectively. Also, the Young's modulus values of carbon/epoxy composite was found as 101,59 GPa, 9,38 GPa and 7,50 GPa in the directions of x (1), y (2) and z (3), respectively as well. According to these data, it was

determined that the maximum values of the Young's modulus for both composite materials were in the x (1) direction and the lowest values were in the z (3) direction.

The Poisson ratios of the materials used in the study are calculated by the elastic coefficients calculated by the equation (2) and the substitution of the D parameter in equation (3) and the data obtained are given in Table 9.

*Table 9. E-glass/epoxy and carbon/epoxy composite Poisson ratio ( $\nu_{12}$ ,  $\nu_{13}$ ,  $\nu_{23}$ ) values*

Composite type		$\nu_{12}$	$\nu_{13}$	$\nu_{23}$
E-glass Epoxy	/	0,17	0,50	0,46
Carbon epoxy	/	0,92	0,11	0,47

As can be seen from Table 9, the lowest Poisson's ratio of e-glass/epoxy composite was found for  $\nu_{12}$  with a value of 0.17 and the highest was found for  $\nu_{13}$  with a value of 0.50. On the other hand, the lowest Poisson's ratio of carbone-glass/epoxy composite was found for  $\nu_{13}$  with a value of 0.11 and the highest value was found for  $\nu_{12}$  with a value of 0.92. The fact that both composites give the lowest and highest Poisson values in different directions shows that these materials show different properties in different directions.

Both composites used in this study have three shear modules in different directions since they are ortotropic in structure. The values of the shear modulus ( $G_{12}$ ,  $G_{13}$ ,

$G_{23}$ ) calculated by substituting the measured densities and ultrasonic velocity values for both materials in equation (2) are given in Table 10.

**Table 10.** *E-glass/epoxy and carbon/epoxy composite for xy, xz and yz plane shear modulus ( $G_{12}$ ,  $G_{13}$ ,  $G_{23}$ ) values*

Composite type		$G_{12}$	$G_{13}$	$G_{23}$
E-glass	/	5,92	3,85	2,92
Epoxy				
Carbon epoxy	/	4,32	4,16	2,54

According to Table 10 data, the shear modulus of the e-glass/epoxy composite was found to be 5,92 GPa in the xy plane, 3,85 GPa in the xz plane and 2,92 GPa in the yz plane. The shear modulus of the carbon/epoxy composite was found as 4,32 GPa in the xy plane, 4,16 GPa in the xz plane and 2,54 GPa in the yz plane. As it can be seen from Table 10, the highest shear modulus in both composite materials is in the xy plane while the lowest was calculated in the yz plane.

#### 4. RESULTS AND DISCUSSIONS

The production of fiber-reinforced composite materials and its use in industrial applications has increased rapidly in recent years. Today, composite materials are preferred to other materials due to their superior resistance to external conditions such as lightness, corrosion resistance and abrasion resistance. Carbon and glass fiber reinforced polymer composites are

widely used in the aerospace, automotive, maritime, transportation and construction sectors as well as in the military field. In this way, the analysis of the mechanical properties and quality control of the materials used in important areas is extremely important before the usage of them.

Calculations of elasticity coefficients by destructive tests such as tensile-compression is so difficult, time-consuming and not economically as well. In addition, the calculation of the mechanical properties is very difficult and requires complex mathematical calculations. However, in order to determine the elasticity coefficients by ultrasonic method, it is sufficient to measure ultrasonic velocities and density in the examined sample. The measurement results can be calculated more precisely because it is possible to perform as many measurements as necessary in the ultrasonic test. As the materials used in this study are prepared by cutting in different directions as given in Table 2, it may seem like a destructive test. However, the speed of the ultrasonic velocity measurements can be repeated as much as desired in the prepared samples and these materials do not cause any damage. On the other hand, the elasticity coefficients can also be measured with only one sample without completely cutting the materials in different directions. In this case, the elasticity coefficients can be measured in the desired direction by using the ultrasonic immersion method. For this measurement method, the examined sample should be only rotated in desired direction in the immersion liquid and only the ultrasonic longitudinal and shear wave velocities in different directions should be measured.

Mistou et al. (1999) have measured the elasticity coefficients of glass fiber reinforced glass/polyester composites by three different techniques (destructive, cutting the sample in the different directions using

ultrasonic method and without cutting the sample but rotating in the immersion liquid method). They have compared the data obtained from these three different methods and they figured out that the ultrasonic method can be used very successfully, quickly and easily. For this reason, in this research the mechanical properties of e-glass/epoxy and carbon/epoxy composites such as elastic modulus, Young's modulus, Poisson's ratios and shear modulus are calculated by using ultrasonic method and the results are discussed below.

According to Table 5, the longitudinal ultrasonic wave velocities for different directions were found as  $x(1) > y(2) > z(3)$  for both the e-glass/epoxy and the carbon/epoxy composite. The data obtained as a result of the combustion test reveal the reason for this ranking. As a result of the combustion test, it was seen that the fiber orientation was the maximum in  $x(1)$ , then  $y(2)$  direction in both materials and in the  $z(3)$  direction there was no reinforcement of the fiber. This result shows the reason for the difference between the longitudinal velocity values. When these two results are compared, it is seen that there is an accurate ratio between the amount of fiber reinforced in the same directions as the longitudinal ultrasonic wave velocity in the  $x$ ,  $y$  and  $z$  directions.

When the ultrasonic wave velocities propagating in the  $x$ ,  $y$  and  $z$  directions given in Table 5 are examined, it is seen that the highest velocity in both materials is obtained in  $V_{12}$ ,  $V_{13}$  and  $V_{23}$ , respectively. When the shear ultrasonic wave velocities ( $V_{12}$  and  $V_{13}$ ) propagating in the direction of  $x(1)$  compared, it can be seen that  $V_{13}$  is lower than  $V_{12}$ . It can be stated that the reason of this, is that there was not found any fiber reinforcement in direction of  $z(3)$ . and the transverse direction of the transverse ultrasound is 3 and the transverse ultrasound with the direction of propagation 2 direction of  $V_{13}$

compared to  $V_{23}$  and  $V_{32}$  in the same direction. It can be said that the low velocity of the wave velocity is due to the fact that the fibers in the direction y (2) are less than the amount of fibers in the x (1) direction. When the  $V_{13}$  and  $V_{23}$  velocities which are vibrating in the same direction z(3) are compared. It is seen that the  $V_{23}$  is lower than  $V_{13}$  due to the fact that the fiber in the direction of y (2) is less than the amount of fiber in direction x (1).

When the ultrasonic wave velocities propagated in the planes cut at  $45^\circ$  angle of composite materials are examined in Table 5, it is seen that the highest velocity in both materials is measured in  $V_{s3}$ , then in  $V_{s2}$  and in  $V_{s1}$  at the lowest speed. According to the data in Table 2,  $V_{s3}$  shear ultrasonic wave velocity measurement is made from the plane obtained by cutting the composite material with xy axes at an angle of  $45^\circ$ ,  $V_{s2}$  ultrasonic wave velocity measurement is made from the plane obtained by cutting the composite material with xz axes at  $45^\circ$  angle and  $V_{s1}$  ultrasonic wave velocity measurement is seen from the plane obtained by cutting the composite material at an angle of  $45^\circ$  with yz axes. According to this result, it can be said that the maximum fiber orientation is measured in the plane where the  $V_{s3}$  ultrasonic wave velocity is measured, then in the plane where the  $V_{s2}$  ultrasound wave velocity is measured and the least fiber orientation is in the plane in which the  $V_{s1}$  ultrasound wave velocity is measured. These data also show that fiber orientation increases the ultrasonic wave velocities. According to the general result obtained from these data, both longitudinal and shear ultrasonic wave velocities increase in proportion to the amount of fiber.

Table 8 also shows that the Young's modulus ( $E_1$ ,  $E_2$  and  $E_3$ ) of orthotropic materials depend on longitudinal and transverse ultrasound wave velocities. On the other hand, it can be said that longitudinal ultrasound wave

velocities affect the Young's modulus value in the direction in which it is measured. This means that if the value of the longitudinal ultrasonic wave velocity is high, the Young's modulus ( $E$ ) value is high and when it is lower, the Young's modulus is lower as well. It was also stated that there was an accurate ratio between the amount of fibers reinforced in the same directions as the longitudinal ultrasonic wave velocity in the  $x$ ,  $y$  and  $z$  directions. In this respect, in both materials the Young's modulus ( $E_1$ ) is high in the direction ( $0^\circ$ ), where the fiber reinforcement is high, the Young's modulus ( $E_2$ ) in the direction ( $90^\circ$ ), where the fiber reinforcement is low and the Young's modulus ( $E_3$ ) in the direction of  $z(3)$ , where there is not any fiber reinforcement used is measured as the lowest. From the data in Table 8, it can be stated that the Young's modulus has a high value in the fiber direction and a low value in the direction perpendicular to the fiber plane. From these data it can be deduced that the Young's modulus has changed according to the fiber orientation.

Since the composite materials with orthotropic properties show different behaviors in different directions depending on their fiber orientation, the strength of the object is higher in the direction that Young's modulus has the highest value. This shows that the object can withstand higher tension and pressure in this direction. Also, the deformation of the substance is easier in the direction which Young's modulus is lower. The increase in Young's modulus shows that atomic bonds are strong at the atomic level, while the decrease of Young's modulus indicates that interatomic bonds are weakened. This data is very important in terms of demonstrating that the information about the binding properties of solids and atoms can be obtained.

As can be seen from Table 8, the Young's modulus of the epoxy resin, which constitutes the matrix of both



composites, is measured as 3,5 GPa. The highest value of the Young modulus of the E-glass/epoxy composite, which obtained by the addition of E-glass fiber has the Young's modulus of 73 GPa, is measured as 33,51 GPa. The highest Young's modulus of the carbon/epoxy composite, which is made with addition of carbon fibers have Young's modulus value of 240 GPa, is measured as 101,59 GPa. Thus, the calculated Young's modulus values of both composite materials were found among the Young's modulus values of the matrix forming the material and values of fibers. Therefore, it can be indicate that the strength of the epoxy matrix which have low Young's modulus, increases when e-glass and carbon fiber are reinforced.

When the carbon/epoxy and e-glass/epoxy composites' Young's modules obtained in all three directions are proportioned in x (1), y (2) and z (3) directions, the values of 3.03, 0.63 and 1.07 is determined. The reason for the fact that the ratio is less than 1 is due to the very low reinforcement of carbon fibers in the carbon/epoxy composite. It is seen that Young's modulus values are approximately equal in 3 directions but three times in the direction of 1. In the direction of x (1), the ratio of carbon fiber-reinforced composite is higher than e-glass fiber reinforced composite (Table 8).

The research results of Marques and Williams (1986) supports the results obtained for the glass/epoxy composites in this research. They have calculated the unidirectional glass fiber/epoxy composite's Young's modulus as 37,76 GPa ( $E_z$ ) in fibers direction and as 8,79 GPa ( $E_x$ ) in direction perpendicular to fiber plane. In this research, the e-glass/epoxy composite's Young's modulus is calculated as 33,51 GPa ( $E_1$ ) in fibers direction and as 7,00 GPa ( $E_3$ ) in direction perpendicular to fiber plane. In his study, Smith (2001) measured

Young's modulus of carbon fiber epoxy composite material containing 62% of carbon fiber as 145 GPa in the fiber direction ( $0^0$ ) and as 9,4 GPa in direction perpendicular( $90^0$ ) to the plane of the fiber(Smith, 2001). In this study, Young's modulus was found as 101,59 GPa (E1) in fiber direction and as 7,50 GPa (E3) in direction perpendicular to fiber plane in carbon epoxy composite. Thus, the results of the study by Smith (2001) support Young's modulus values obtained in this study for carbon/epoxy composite sample.

According to Table 9 data, the lowest Poisson's ratio of e-glass/epoxy composite was found as 0,17 for  $\nu_{12}$  and the highest as 0,50 for  $\nu_{13}$ . The lowest Poisson ratio for carbon/epoxy composite was obtained as 0,11 for  $\nu_{13}$  and the highest value was obtained as 0,92 for  $\nu_{12}$ . An isotropic material has generally a Poisson's ratio of maximum 0,50(Lempriere, 1968). However, the values such as 0,50 and 0,92 are obtained in this research. This is due to structure of orthotropic materials. In the research conducted byOzturk and Erdogan (1997), it was stated that the Poisson's ratio in orthotropic materials could be greater than 0,5. These explanations of Ozturk and Erdogan (1997) support Poisson values higher than 0,50 in this research.

When the shear modulus values given in Table 10 are analyzed, it is seen that the  $G_{12}$  values obtained for both composites are higher than  $G_{13}$  and  $G_{23}$  values. This may be due to the fact that fiber reinforcements in the x (1) direction are more than the other directions. The shear modulus values differ from each other in different directions and are consistent with the results of Young's modulus and shear velocity values which give different values in different directions. Therefore, it can be stated that the addition of the amounts of fiber reinforcement to different amounts of the matrix in different directions

causes the elasticity properties in those directions to be different.

In this study, mechanical properties of orthotropic materials were determined by using ultrasonic pulse-echo technique. The elasticity properties determined by ultrasonic method can be compared with the results obtained by tensile and shear tests. Similar studies can be made to determine the elasticity properties of composite materials or crystals with different symmetry. Also, similar studies can be done by using different fibers have different properties in different ratio.



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# GENERALIZED MULTISET TOPOLOGIES

BÖLÜM  
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## 1. INTRODUCTION

Firstly, Loeb introduced multisets in [4] by assuming that for an arbitrary set, an element occurs a finite number of times. Multisets are used in computer science for quantitative analysis and models of resources. Then the concept of generalized multiset was defined first by Loeb in [4]. The multiplicity of an element in a generalized multiset is either a positive number, zero or a negative number. A. Alexandru and G. Ciobanu [1] have studied on algebraic properties of generalized multisets. K.P. Girish and S.J. John investigated multiset topologies. In this paper, our aim is to extend some results of the paper [3] for generalized multisets and to focus on generalized multiset topologies.

## 2. ON GENERALIZED MULTISSETS

Let  $X = \{x_1, x_2, \dots, x_n\}$ . An generalized multiset  $M$  over  $X$  is a function  $C_M: X \rightarrow \mathbb{Z}$  where  $C_M(x)$  is the multiplicity of  $x \in X$ . We denote the generalized multiset by  $\{ \mid \}$ . In a generalized multiset, elements having positive multicity are written on the left of the bar and elements having negative multicity are written on the right of the bar. Ordering in the set is not important. But we never write the same element on both sides of the bar. Summarizing, generalized multiset  $M$  is written as the following form

$$M = \{k_1|y_1, k_2|y_2, \dots, k_i|y_i|k_{i+1}|y_{i+1}, \dots, k_s|y_s\}$$

where  $y_1, \dots, y_s \in X,$   
 $k_1 = C_M(y_1), k_2 = C_M(y_2), \dots, k_i = C_M(y_i) \geq 0$  and  
 $k_{i+1} = C_M(y_{s+1}), \dots, k_s = C_M(y_s) \leq 0.$

**Example 2.1.**  $X = \{a, b, c, d, e\}$  is any set. If  $M = \{2|a, 1|e|4|b, 5|d\}$ , then  $C_M(a) = 2, C_M(b) = 4, C_M(c) = 0, C_M(e) = 1, C_M(d) = -5$ .

If a generalized multiset has only values 0 and 1, this generalized multiset is called a classical set. If a generalized multiset has only values 0 and -1, we say a negative set. The empty set  $\emptyset = \{ \mid \}$  is the generalized multiset which have multiplicity zero for all elements. By  $\mathbb{Z}(X)$ , we denote the set of all generalized multisets over  $X$ . We define some properties as follows:

- (1)  $M = N$  if  $C_M(x) = C_N(x)$ , for all  $x \in X$ .
- (2)  $M \subseteq N$  if  $|C_M(x)| \leq |C_N(x)|$ , for all  $x \in X$ .
- (3)  $P = M \cup N$  if

$$C_P(x) \begin{cases} \max(C_M(x), C_N(x)), & \text{if } x \text{ appears on the left side in } M \text{ and } N \\ \min(C_M(x), C_N(x)), & \text{if } x \text{ appears on the right side in } M \text{ and } N \\ C_M(x), & \text{if } x \text{ appears on the left in } M \text{ on the right in } N \\ C_N(x), & \text{if } x \text{ appears on the left in } N \text{ on the right in } M \end{cases}$$

- (4)  $P = M \cap N$  if

$$C_P(x) \begin{cases} \min(C_M(x), C_N(x)), & \text{if } x \text{ appears on the left side in } M \text{ and } N \\ \max(C_M(x), C_N(x)), & \text{if } x \text{ appears on the right side in } M \text{ and } N \\ C_N(x), & \text{if } x \text{ appears on the left in } M \text{ on the right in } N \\ C_M(x), & \text{if } x \text{ appears on the left in } N \text{ on the right in } M \end{cases}$$

**Example 2.2.** Let  $M = \{2|a, 3|b|1|c, 2|d\}$  and  $N = \{4|a, 3|b|2|c, 3|d\}$ . It is clear that  $M \subseteq N, M \cup N = \{4|a, 3|b|2|c, 3|d\}, M \cap N = \{2|a, 3|b|1|c, 2|d\}$ .

Let  $M$  be generalized multiset over  $X$ . By  $M^*$ , we denote the support set of  $M$  defined by  $M^* = \{x \in X; |C_M(x)| > 0\}$ . The cardinality of generalized multiset  $M$  is denoted by  $|M|$ . It is given by

$|M| = \sum_{x \in X} |C_M(x)|$ . By  $[M]_x$ , we denote the element  $x$  belonging to the generalized multiset  $M$ . By  $|[M]_x|$ , we denote the cardinality of an element  $x$  in  $M$ . The generalized multiset space  $[X]^m$  is the set of all generalized multisets whose elements are in  $X$  such that  $|C_M(x)| \leq m$ . The complement  $M^c$  of  $M$  in  $[X]^m$  is an element of  $[X]^m$  such that  $C_M^c(x) = m - |C_M(x)|$  for all  $x \in X$ .

**Definition 2.1. (Whole generalized submultiset)** A generalized submultiset  $N$  of  $M$  is a whole generalized submultiset of  $M$  with each element in  $N$  having full multiplicity as in  $M$ ,  $C_N(x) = C_M(x)$  for every  $x$  in  $N^*$ .

**Example 2.3.** Let  $M = \{2|x, 3|y, 5|z|1|p, 2|s\}$ .  $N = \{2|x|2|s\}$  is whole generalized submultiset of  $M$ .

**Definition 2.2. (Partial whole generalized submultiset)** A generalized submultiset  $N$  of  $M$  is a partial whole generalized submultiset of  $M$  with at least one element in  $N$  having full multiplicity as in  $M$ . Namely,  $C_N(x) = C_M(x)$  for some  $x$  in  $N^*$ .

**Example 2.4.** Let  $M = \{2|x, 3|y, 5|z|1|p, 2|s\}$ .  $N = \{1|x, 2|y, 3|z|1|p, 1|s\}$  is partial whole generalized submultiset of  $M$ .

**Definition 2.3. (Full generalized submultiset)** A generalized submultiset  $N$  of  $M$  is a full generalized submultiset of  $M$  if each element in  $M$  is an element in  $N$  with the same or lesser non-zero multiplicity as in  $M$ , that is,  $M^* = N^*$  with  $C_N(x) \leq C_M(x)$  for every  $x$  in  $N^*$ .

**Example 2.5.** Let  $M = \{2|x, 3|y, 5|z|1|p, 2|s\}$ .  $N = \{1|x, 2|y, 3|z|1|p, 1|s\}$  is full generalized submultiset.

**Definition 2.4. (Power whole generalized multiset)**

Let  $M \in [X]^m$  be a generalized multiset. The power whole generalized multiset of  $M$  denoted by  $PW(M)$  is defined as the set of all the whole generalized submultisets of  $M$ . The cardinality of  $PW(M)$  is  $2(2^n - 1)$ , where  $n$  is the cardinality of  $M$ .

**Definition 2.5. (Power full generalized multiset)**

Let  $M \in [X]^m$  be a generalized multiset. The power full generalized multiset of  $M$  denoted by  $PF(M)$  is defined as the set of all the full generalized submultisets of  $M$ . The cardinality of  $PF(M)$  is the product of the counts of the elements in  $M$ .

**Definition 2.6. (Power generalized multiset)**

Let  $M \in [X]^m$  be a generalized multiset. The power generalized multiset of  $M$  denoted by  $P(M)$  is defined as the set of all the generalized submultisets of  $M$ .

The power set of a generalized multiset is the support set of the power generalized multiset and it is denoted by  $P^*(M)$ .

**Example 2.6.** Let  $M = \{2|x|3|y\}$  be a generalized multiset. The set

$$PW(M) = \{\{2|x|\}, \{3|y|\}, M, \emptyset, \{2|x, 3|y|\}, \{2|x, 3|y|\}, \{3|y|2|x|\}, \{2|x\}, \{3|y|\}\}$$

is the power whole generalized submultiset of  $M$ . The set

$$\begin{aligned}
 P(M) = & \{1/\{2|x| \}, 1/\{ |3|y\}, M, \emptyset, 1/\{2|x, 3|y| \}, 1 \\
 & / \{ |2|x, 3 |y\}, 1/\{3|y|2 |x\}, 1 \\
 & / \{ |2|x\}, \{3|y|\}, 2/\{1|x| \}, 2/\{1|x|3|y\}, 2 \\
 & / \{1|x, 3|y| \}, 2/\{1,3|y\}, \{2/\{3|y|1|x\}, 2 \\
 & / \{1|x\}, 3/\{2|x|2|y\}, 3/\{2|x, 2|y|\}, 3 \\
 & / \{2|x, 2|y\}, 3/\{2|y|2|x\}, 3/\{2|y\}, 3 \\
 & / \{2|y|\}, 6/\{1|x|2|y\}, 6/\{1|x, 2|y|\}, 6 \\
 & / \{1|x, 2|y\}, 6/\{2|y, 1|x|\}, 6/\{1|x|2|y\}, 3 \\
 & / \{1|y|\}, 6/\{1|x|1|y\}, 6/\{1|x, 1|y|\}, 6 \\
 & / \{1|x, 1|y\}, 6/\{1|y|1|x\}, 3/\{1|y\}, 3 \\
 & / \{2|x|1|y\}, 3/\{2|x, 1|y|\}, 3/\{2|x, 1|y\}, 3 \\
 & / \{1|y|2|x\}\}
 \end{aligned}$$

is the power generalized submultiset of  $M$ .

### 3. GENERALIZED MULTISSET TOPOLOGIES

**Definition 3.1. (Operations under collection of generalized multisets)** Let  $[X]^m$  be a generalized multiset space and  $\{M_1, M_2, \dots\}$  be a collection of generalized multisets from  $[X]^m$ . Then

(i) The union  $\bigcup_{i \in I} M_i = \{C_{\bigcup M_i}(x) / x \mid C_{\bigcup M_i}(x) = \max\{C_{M_i}(x) \mid x \in X\}\}$ .

(ii) The intersection  $\bigcap_{i \in I} M_i = \{C_{\bigcap M_i}(x) / x \mid C_{\bigcap M_i}(x) = \min\{C_{M_i}(x) \mid x \in X\}\}$ .

**Definition 3.2.** Let  $M \in [X]^m$  and  $\tau \subseteq P^*(M)$ . If  $\tau$  satisfies

(i)  $\emptyset$  and  $M$  are in  $\tau$

(ii) The union of the elements of any subcollection of  $\tau$  is again in  $\tau$

(iii) The intersection of the elements of any finite subcollection of  $\tau$  is again in  $\tau$ ,

then  $\tau$  is called a generalized multiset topology.

**Example 3.1.** The collection consisting of only  $M$  and  $\emptyset$  is an  $M$ -topology called trivial  $M$ -topology.

**Example 3.2.** The collection  $PW(M)$  is an  $M$ -topology on  $M$ .

**Example 3.3.** Since  $\emptyset$  is not in  $PF(M)$ , the collection  $PF(M)$  is not an  $M$ -topology but  $PF(M) \cup \{\emptyset\}$  is an  $M$ -topology on  $M$ .

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**ON THE EXPANSION  
FORMULA FOR A  
SINGULAR STURM-  
LIOUVILLE  
OPERATOR**

**BÖLÜM  
6**

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## 1. Introduction

In this study, the boundary value problem generated by the Sturm-Liouville equation

$$\ell(y) \equiv -y'' + q(x)y = \lambda^2 y \quad (1.1)$$

and the boundary condition

$$y'(0) + (\alpha_0 + i\alpha_1\lambda + \alpha_2\lambda^2)y(0) = 0 \quad (1.2)$$

have been investigated in the space  $L_2[0, \infty)$ . Here  $\lambda$  is a complex parameter,  $q(x)$  is a real valued function satisfying the condition

$$\int_0^{\infty} (1+x)|q(x)| dx < \infty, \quad (1.3)$$

and  $\alpha_i (i=0,1,2)$  are real numbers such that  $\alpha_1 > 0, \alpha_2 > 0$ .

Denote

$$D(L_\lambda) = \left\{ \begin{array}{l} y(x) \mid y(x), y'(x) \in AC([a,b]) \text{ for all } b > 0, \\ \ell(y) \in L_2(0, \infty), y'(0) + (\alpha_0 + i\alpha_1\lambda + \alpha_2\lambda^2)y(0) = 0 \end{array} \right\}$$

Here, by  $AC([a,b])$  we denote the set of all absolutely continuous functions on the interval  $[a,b]$ .

Let  $L_{\lambda_0}$  is operator with domain  $D_{\lambda_0} = D(L_{\lambda_0})$  such that for  $y \in D_{\lambda_0}$  satisfies  $L_{\lambda_0} y = \ell(y)$ . If  $\lambda$  runs

through the set off all points of the  $\lambda$ -plane, then we obtain a family of singular operators  $L_x$  depending on the parameter  $\lambda$  (see [1], [2],[4]). Similar problems have been investigated in mathematical physics and they are called Regge problems in literature (See [3], [8] and others). In non-adjoint cases, for the singular Sturm-Liouville problem the spectral expansion problem are studied in [1], [2], [7].

Denote by  $e(x, \lambda)$  the Jost solution of the equation (1.1) possessing the asymptotics  $\text{Im } \lambda \geq 0$

$$\lim_{x \rightarrow \infty} e(x, \lambda) e^{-i\lambda x} = 1.$$

It is well known from [6] for any  $\lambda$  from the closed half plane  $\text{Im } \lambda \geq 0$ , the equation (1.1) has a solution  $e(x, \lambda)$  that can be represented in the form

$$e(x, \lambda) = e^{i\lambda x} + \int_x^\infty K(x, t) \exp\{i\lambda t\} dt \quad (1.4)$$

The kernel  $K(x, t)$  satisfies the inequality

$$K(x, t) \leq \frac{1}{2} \sigma\left(\frac{x+t}{2}\right) \exp\left\{\sigma_1(x) - \sigma_1\left(\frac{x+t}{2}\right)\right\}$$

where  $\sigma(x), \sigma_1(x)$  are defined by the following formulas

$$\sigma(x) = \int_x^\infty |q(t)| dt, \quad \sigma_1(x) = \int_x^\infty \sigma(t) dt.$$

Moreover, the function  $e(x, \lambda)$  has the some properties in  $\text{Im } \lambda \geq 0$  (see [6]).

For all real  $\lambda \neq 0$  the functions  $e(x, \lambda)$  and  $e(x, -\lambda)$  form a fundamental system of solutions of equation (1.1) and their Wronskian is

$$W\{e(x, \lambda), e(x, -\lambda)\} = e'(x, \lambda)e(x, -\lambda) - e(x, \lambda)e'(x, -\lambda) = 2i\lambda.$$

Let  $\omega(x, \lambda)$  be a special solution of the equation (1.1) satisfying the following initial-value conditions

$$\omega(0, \lambda) = 1, \omega'(0, \lambda) = -(\alpha_0 + i\alpha_1\lambda + \alpha_2\lambda^2).$$

Analogously to the Lemma 1 in [5] is just like that

$$\frac{2i\lambda\omega(x, \lambda)}{E(\lambda)} = e(x, -\lambda) - S(\lambda)e(x, \lambda), \quad (1.5)$$

where

$$S(\lambda) = \frac{E_1(\lambda)}{E(\lambda)}, \quad (1.6)$$

$$E(\lambda) = e'(0, \lambda) + (\alpha_0 + i\alpha_1\lambda + \alpha_2\lambda^2)e(0, \lambda),$$

$$E_1(\lambda) = e'(0, -\lambda) + (\alpha_0 - i\alpha_1\lambda + \alpha_2\lambda^2)e(0, -\lambda).$$

The function  $S(\lambda)$  is called the scattering function for the boundary value problem (1.1), (1.2). It is clearly, the function  $S(\lambda)$  is meromorphic in the half plane  $\text{Im}\lambda > 0$ , with poles at the zeros of the function  $E(\lambda)$ . The function  $E(\lambda)$  is analytic in the upper half plane and for the scattering data function  $S(\lambda)$  the asymptotic formula hold as  $|\lambda| \rightarrow \infty, S(\lambda) = 1 + O\left(\frac{1}{\lambda}\right)$ .

We denote by  $\lambda_k$  the roots of the equation  $E(\lambda) = 0$  in the half plane  $\text{Im } \lambda > 0$ . Using the form (1.3) of the Jost solution  $e(x, \lambda)$ , it can be proved that  $E(\lambda)$  may have only a finite number of zeros in the half plane  $\text{Im } \lambda > 0$ . Moreover, all these zeros are simple and lie on the imaginary axis. For  $\lambda = i\lambda_k$  ( $\lambda_k > 0$ ), ( $k = 1, 2, \dots, n$ ) we get

$$\begin{aligned}
 m_k^{-2} &= \int_0^\infty |e(x, i\lambda_k)|^2 dx + \frac{\alpha_1 + 2\alpha_2 \lambda_k}{2\lambda_k} |e(0, i\lambda_k)|^2 \\
 &= \frac{-i\dot{E}(i\lambda_k)e(0, i\lambda_k)}{2\lambda_k}.
 \end{aligned}
 \tag{1.7}$$

## 2. Resolvent Operator

Assume that  $\lambda$  is not a spectrum point of the operator  $L(\lambda)$ . Let us find the expression of the resolvent operator  $R_\lambda = (L - \lambda I)^{-1}$ .

**Theorem 2.1.** The resolvent  $R_\lambda$  is the integral operator

$$R_\lambda = \int_0^\infty R(x, t, \lambda) f(t) dt,
 \tag{2.1}$$

with the kernel

$$R(x, t, \lambda) = -\frac{1}{E(\lambda)} \begin{cases} e(x, \lambda)\omega(t, \lambda), & x \leq t \\ \omega(x, \lambda)e(t, \lambda) & t \leq x \end{cases}
 \tag{2.2}$$

also

$$|R(x,t,\lambda)| \leq \frac{C(x)}{E(\lambda)} \exp\{-\text{Im } \lambda |x-t|\}, \quad (2.3)$$

where

$$C(x) = c \cdot \exp\{x\sigma(0) + \sigma_1(0)\}, \quad c = \text{const.}$$

**Proof:** To construct the resolvent operator  $R_\lambda$ , let us investigate the solution  $y(x, \lambda)$  of the equation

$$-y'' + q(x)y = \lambda^2 y + f(x)$$

satisfying the condition

$$y'(0) + (\alpha_0 + i\alpha_1\lambda + \alpha_2\lambda^2)y(0) = 0$$

where  $f(x) \in D_\lambda$  is zero in exterior of every interval. If we apply the Lagrange method by using properties solutions of the equation (1.1), we find (2.1) where  $R(x,t,\lambda)$  is in the form of (2.2).

From the explicit expression of functions  $e(x, \lambda)$  and  $\omega(x, \lambda)$  it follows that (2.3). Theorem is proved.

It is clear that every  $\lambda$  satisfying the condition  $E(\lambda) \neq 0$  belongs to the resolvent set of the operator  $L_\lambda$ .

The resolvent  $R_\lambda = (L - \lambda I)^{-1}$  is an integral operator of the form (2.1). The kernel  $R(x,t,\lambda)$  satisfies the inequality (2.3).

**Lemma 2.2.** Let the function  $f(x)$  has a continuous second derivative and vanish outside the finite intervals. Then, the following is valid as  $|\lambda| \rightarrow \infty, \text{Im } \lambda \geq 0$



$$\int_0^{\infty} R(x, t, \lambda) f(t) dt = -\frac{f(x)}{\lambda^2} + \frac{1}{\lambda^2} \int_0^{\infty} R(x, t, \lambda) \tilde{f}(t) dt, \quad (2.4)$$

where

$$\tilde{f}(t) = -f''(t) + q(t)f(t).$$

**Proof:**

The below equality is valid

$$-R''(x, t, \lambda) + q(x)R(x, t, \lambda) - \lambda^2 R(x, t, \lambda) = \delta(x - t).$$

Multiplying both sides of this equation by  $f(x)$  and integrating from zero to infinity, and using property of the delta function  $\delta$  and partial differentiation, we obtain

$$-\int_0^{\infty} R(x, t, \lambda) f''(t) dt + \int_0^{\infty} q(t)R(x, t, \lambda) f(t) dt - \lambda^2 \int_0^{\infty} R(x, t, \lambda) f(t) dt = f(x)$$

From this equation, we obtain (2.4). Lemma is proved.

### 3. The Expansion Formula

Put

$$F(x, \lambda) = \int_0^{\infty} R(x, t, \lambda) f(t) dt.$$

By  $\Gamma_R$  we denote the circle of radius  $R$  and center is zero which contour is positive oriented. Let us  $\Gamma_{R,\varepsilon}^{(1)}$

denotes contour be half arc of  $\Gamma_R$  that doesn'tt include points  $z$  satisfying

the conditions  $\text{Im } z < \varepsilon$ . Let  $\Gamma_{R,\varepsilon}^{(2)}$  by half arc that does not include  $\text{Im } z > -\varepsilon$  points of  $\Gamma_R$  and

we defined  $\Gamma_{R,\varepsilon} = \Gamma_{R,\varepsilon}^{(1)} \cup \Gamma_{R,\varepsilon}^{(2)}$  it is clearly,  $\Gamma_{R,\varepsilon}$  is positive oriented.  $\Gamma_{R,\varepsilon}^{(3)}$  denotes be a negative oriented curve formed with  $\text{Im } z = \pm\varepsilon$  lines and be arcs including points  $z$  satisfying the conditions  $|\text{Im } z| \leq \varepsilon$ .

According to this,  $\Gamma_{R,\varepsilon} = \Gamma_R \cup \Gamma_{R,\varepsilon}^{(3)}$ . Then we can use property of the integration

$$\int_{\Gamma_{R,\varepsilon}} = \int_{\Gamma_R} + \int_{\Gamma_{R,\varepsilon}^{(3)}} . \tag{3.1}$$

Now multiplying both sides of equality (2.4) by  $\frac{\lambda}{2\pi i}$

and integrating over  $\frac{\lambda}{2\pi i}$  the contour  $\Gamma_{R,\varepsilon} = \Gamma_R \cup \Gamma_{R,\varepsilon}^{(3)}$ , we obtain

$$\begin{aligned} & \frac{1}{2\pi i} \int_{\Gamma_{R,\varepsilon}} \lambda F(x, \lambda) d\lambda = \\ & -\frac{1}{2\pi i} \int_{\Gamma_{R,\varepsilon}} \frac{f(x)}{\lambda} d\lambda + \frac{1}{2\pi i} \int_{\Gamma_{R,\varepsilon}} \frac{1}{\lambda} \left\{ \int_0^\infty R(x, t, \lambda \tilde{f}(t)) \right\} d\lambda \end{aligned} \tag{3.2}$$

where

$$\tilde{f}(t) = -f''(t) + q(t)f(t).$$

According to the equation (3.1) we get

$$\frac{1}{2\pi i} \int_{\Gamma_{R,\varepsilon}} \lambda F(x, \lambda) d\lambda = \tag{3.3}$$

$$\frac{1}{2\pi i} \int_{\Gamma_R} \lambda F(x, \lambda) d\lambda + \frac{1}{2\pi i} \int_{\Gamma_{R,\varepsilon}^{(3)}} \lambda F(x, \lambda) d\lambda.$$

From (3.2) we have

$$\lim_{R \rightarrow \infty} \frac{1}{2\pi i} \int_{\Gamma_R} \lambda F(x, \lambda) d\lambda = -f(x), \tag{3.4}$$

and

$$\lim_{\substack{R \rightarrow \infty \\ \varepsilon \rightarrow 0}} \frac{1}{2\pi i} \int_{\Gamma_{R,\varepsilon}} \lambda F(x, \lambda) d\lambda = \tag{3.5}$$

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} \lambda [F(x, \lambda + i0) - F(x, \lambda - i0)] d\lambda.$$

On the other hand using the residue calculus we get

$$\frac{1}{2\pi i} \int_{\Gamma_{R,\varepsilon}} \lambda F(x, \lambda) d\lambda = \sum_{j=1}^n \operatorname{Res}_{\lambda=i\lambda_j} [\lambda F(x, \lambda)] + \sum_{j=1}^n \operatorname{Res}_{\lambda=-i\lambda_j} [\lambda F(x, \lambda)].$$

From the relations (3.3), (3.4), (3.5) we write

$$\frac{1}{2\pi i} \int_{\Gamma_{R,\varepsilon}} \lambda F(x, \lambda) d\lambda = -f(x) + \frac{1}{2\pi i} \int_{-\infty}^{\infty} \lambda [F(x, \lambda + i0) - F(x, \lambda - i0)] d\lambda$$

From the last relation we get

$$\begin{aligned}
 f(x) = & -\sum_{j=1}^n \operatorname{Res}[\lambda F(x, \lambda)] - \sum_{j=1}^n \operatorname{Res}[\lambda F(x, \lambda)] \\
 & + \int_{-\infty}^{\infty} \lambda [\lambda F(x, \lambda + i0) - \lambda F(x, \lambda - i0)] d\lambda. \quad (3.6)
 \end{aligned}$$

Let  $\psi(x, \lambda)$  be solution of the equation (1.1) satisfying the initial conditions

$$\psi(0, \lambda) = 0, \quad \psi'(x, \lambda) = -\alpha_1.$$

Then, for  $\operatorname{Im} \lambda \geq 0$  we have

$$e(x, \lambda) = c_1 \omega(x, \lambda) + c_2 \psi(x, \lambda),$$

Where

$$c_1 = e(0, \lambda), \quad c_2 = -\frac{E(\lambda)}{\alpha_1}.$$

Therefore

$$e(x, \lambda) = e(0, \lambda) \omega(x, \lambda) - \frac{E(\lambda)}{\alpha_1} \psi(x, \lambda),$$

and

$$\begin{aligned}
 R(x, t, \lambda) = & -\frac{1}{E(\lambda)} e(0, \lambda) \omega(x, \lambda) \omega(t, \lambda) \\
 & + \frac{1}{\alpha_1} \begin{cases} \psi(x, \lambda) \omega(t, \lambda), & x \leq t, \\ \psi(t, \lambda) \omega(x, \lambda), & t \leq x. \end{cases}
 \end{aligned}$$

Then, for  $\operatorname{Im} \lambda > 0$  we obtain

$$\begin{aligned}
 F(x, \lambda) = & -\frac{1}{E(\lambda)} e(0, \lambda) \omega(x, \lambda) \int_0^\infty \omega(t, \lambda) f(t) dt + \\
 & \frac{\psi(x, \lambda)}{\alpha_1} \int_0^x \omega(t, \lambda) f(t) dt + \frac{\omega(x, \lambda)}{\alpha_1} \int_x^\infty \psi(t, \lambda) f(t) dt.
 \end{aligned}
 \tag{3.7}$$

Since

$$\operatorname{Re} s_{\lambda=i\lambda_j} \left\{ \frac{\psi(x, \lambda)}{\alpha_1} \int_0^x \omega(t, \lambda) f(t) dt + \frac{\omega(x, \lambda)}{\alpha_1} \int_x^\infty \psi(t, \lambda) f(t) dt \right\} = 0,$$

we have

$$\begin{aligned}
 \operatorname{Re} s_{\lambda=i\lambda_j} [F(x, \lambda)] = \\
 -\frac{i\lambda_j}{\dot{E}(i\lambda_j)} e(0, i\lambda_j) \omega(x, i\lambda_j) \int_0^\infty \omega(t, i\lambda_j) f(t) dt.
 \end{aligned}
 \tag{3.8}$$

By the equalities (1.6) and (3.8) we get

$$\begin{aligned}
 \sum_{j=1}^n \operatorname{Re} s_{\lambda=i\lambda_j} [\lambda F(x, \lambda)] + \sum_{j=1}^n \operatorname{Re} s_{\lambda=-i\lambda_j} [\lambda F(x, \lambda)] = \\
 \sum_{j=1}^n u(x, i\lambda_j) \int_0^\infty u(x, i\lambda_j) f(t) dt,
 \end{aligned}
 \tag{3.9}$$

where

$$u(x, i\lambda_j) = m_j u(x, i\lambda_j), \quad m_j^2 = \frac{2i\lambda_j}{\dot{E}(i\lambda_j) e(0, i\lambda_j)}.$$

Now let us calculate

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} \lambda [F(x, \lambda + i0) - F(x, \lambda - i0)] d\lambda.$$

From the formula (3.7) and the equality  $F(x, \lambda - i0) = \overline{F(x, \lambda + i0)}$ , we have

$$\begin{aligned} F(x, \lambda + i0) - F(x, \lambda - i0) = \\ \left[ -\frac{e(0, \lambda)}{E(\lambda)} + \overline{\frac{e(0, \lambda)}{E(\lambda)}} \right] \omega(x, \lambda) \int_0^{\infty} \omega(t, \lambda) f(t) dt = \\ -\frac{2i\lambda\alpha_1}{|E(\lambda)|^2} |e(0, \lambda)|^2 \omega(x, \lambda) \int_0^{\infty} \omega(t, \lambda) f(t) dt. \end{aligned}$$

It follows that

$$\begin{aligned} \frac{1}{2\pi i} \int_{-\infty}^{\infty} \lambda [F(x, \lambda + i0) - F(x, \lambda - i0)] d\lambda = \\ -\frac{2\alpha_1}{\pi} \int_0^{\infty} \frac{\lambda^2}{|E(\lambda)|^2} \omega(x, \lambda) \int_0^{\infty} \omega(t, \lambda) f(t) dt d\lambda = \\ \frac{\alpha_1}{2\pi} \int_0^{\infty} \frac{i\lambda\omega(x, \lambda)}{E(\lambda)} \int_0^{\infty} \frac{2i\lambda}{E(\lambda)} \omega(t, \lambda) f(t) dt d\lambda. \end{aligned} \tag{3.10}$$

Using (3.9) and (3.10) we get the expansion formula with respect to eigenfunctions as

$$\begin{aligned} f(x) = \sum_{j=1}^n u(x, i\lambda_j) \int_0^{\infty} u(t, i\lambda_j) f(t) dt + \\ \frac{\alpha_1}{\pi} \int_0^{\infty} u(x, \lambda) \int_0^{\infty} \overline{u(t, \lambda)} f(t) dt, \end{aligned} \tag{3.11}$$

where

$$u(x, \lambda) = \frac{2i\lambda}{\omega(x, \lambda)} = \overline{e(x, \lambda)} - S(\lambda)e(x, \lambda).$$

The equalities (3.6) and (3.11) are obtained from eigenfunctions and generalized eigenfunctions, their asymptotic behaviour as  $x \rightarrow \infty$  is determined by the scattering data

$$\{S(\lambda) (-\infty < \lambda < \infty); -\lambda_1^2, -\lambda_2^2, \dots, -\lambda_n^2; m_1, m_2, \dots, m_n\}$$

of the boundary value problem(1.1), (1.2).

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**DEVELOPMENT OF AN  
ANALYTICAL MODEL  
FOR MEASURING  
TECHNICAL EDUCATION  
COMPETENCY USING ANP  
AND TOPSIS**

**BÖLÜM  
7**

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## **Introduction**

The traces, evidence and findings of humans in the past show that human beings were constantly striving to improve the quality of life. In order to maintain and facilitate their lives as well as improve their quality of life, they tried to produce what was not provided by nature. They discovered that the things except what nature invention and discovery made it possible for them to acquire what was not provided by nature thus maintained their lives. Although the first inventions of humans on this planet are seen as very simple compared to those of today's, today's inventions were realized as a result of the accumulation of the past. However, this accumulation was achieved thanks to the education that humans invented again as a means of maintaining their existence on this planet and understanding the nature and universe in which they exist. In other words, education activity adopted a role in the making of these inventions and discoveries primary to maintain their life and developing technologies to bring these to life.

Although the term technology is defined in different ways, it is essentially defined as the systematic application efficiency of the information produced in the process or procedures of producing goods and services. In other words, the concept of technology is defined as a set of knowledge and skills that enable efficient and productive activities involving research, development, production, distribution, marketing and after-sales services (Zerenler, Türker and Şahin, 2007). From a functional point of view, it can be said that technology is the application of science (Günay, 2002). Technology is the activity of using theoretical knowledge into action

and solving problems (Alpaslan, 2011). In addition to being a set of information that enables product design and production, technology is described as all physical processes and arrangements that convert inputs into outputs (Erdil and Pamukçu, 2015). When the concept of technology is examined in terms of economics, technology provides competitive advantage by creating efficient production through the creation of differences in production factors and methods. (Malatyalı, 2016). The activities included in the definition of technology are indispensable for human life. However, the continuation of these activities is also vital for humans. (Günay and Çalık, 2019). For this reason, institutions providing VTE at various academic and application levels have been established in order to facilitate human life and improve the quality of life. One of them is VSHEs which provide knowledge and skills to technicians. Such schools play an important role in all societies or countries with different levels of development. Especially today's rapidly changing scientific and technological advances, as in other areas of life, question the appropriateness or competency of VTE. This requires an examination and investigation of the issue of the competency of VTE.

Although the competency of VTE varies according to the development level of the countries in terms of scope and purpose, the main issue is essentially the provision of VTE competency. In other words, it is seen that countries attach importance to ensuring the significance and competency of VTE. It is seen that they have developed and implemented various VTE models for this purpose (Balcı, Çelik and Eldem, 2013). The significant difference in the level of VTE in developed and

industrialized countries compared to underdeveloped and developing countries (Peran and Bilir, 2007) can be explained by the intensity of production and research and development (R&D) studies (Ünal and Seçilmiş, 2013; Erdil and Pamukçu, 2015). In other words, the advanced knowledge of developed countries in all areas of life can be found in VTE. Nevertheless, developed countries try to maintain their competency in VTE as well as R&D activities and try to increase the level of VTE further. For this reason, scientific studies are carried out for VTE and they are tried to be implemented. In the literature, it is seen that various scientific studies on the competency of VTE are the subject of research (Adıgüzel and Berk, 2009; Karabulut and Marul, 2011; Sahu, Shrivastava, and Shrivastava, 2013; Tuncer and Tanaş, 2019; Özer, 2019; Eskandaripour, Hajhosseinnezdah, Alastair, and Hosseinkhah, 2019).

Although the level of development of the countries varies, the objectives or issues of VTE show similarities (Ömer, 2019), but the main issue is the continuous improvement of VTE competency. In fact, this situation was emphasized in the aims of the studies in the literature (Özer, 2019). In the relevant literature review, there are some unresolved issues about measuring and evaluating the competency of VTE. As with multiple-criteria problems (Cheng and Li, 2007; Chen and Wang, 2010; Karahalios, 2017), the nature of the VTE competency issue requires the analysis of a number of elements and the consideration of the competency of VTE on this basis. As a matter of fact, the findings of the studies in the literature (Tuncer and Tanaş, 2019; Eskandaripour, Hajhosseinnezdah, Aliasgarı, and Hosseinkhah, 2019)

showed that the competency of VTE was related to many factors or was influenced by many factors. However, it is important to note that the importance of these factors in VTE competency may vary. Therefore, it is necessary to consider the differences in the importance of the factors in determining the competency of VTE. The extent to which each of the factors determining the competency of VTE is involved in the competency of VTE will be able to be ascertained by such an assessment. In the related literature, one study that prioritized the competency factors of VTE (Eskandaripour, Hajihosseinnezdah, Aliasgari, and Hossemikhah, 2019) was found. In the study (Eskandaripour, Hajihosseinnezdah, Aliasgari, and Hossemikhah, 2019) prioritization was made with analytical hierarchy process (AHP) technique. However, AHP technique is not suitable for prioritizing more than nine criteria. On the other hand, theoretically and practically, it can be said that VTE competency factors are numerous. This limitation is overcome by grouping the VTE factors in AHP. However, this approach does not solve the problem in some cases. For example, in cases where there is a problem of homogeneity in terms of content of VTE competency factors, grouping cannot be a solution. Another problem encountered in AHP technique is the scope of prioritization. Prioritization in AHP are made in the context of binary comparisons within groups of factors. In other words, in AHP, the relative importance levels of each factor are calculated with the prioritization process (Saaty, 1980). Weighting factors with such an approach may be appropriate for solving some hierarchical problems (Kim, Jang, and Lee, 2013; Chen and Wang, 2010; Dweiri, Kumar, Khan, and Jain, 2016). In the calculation of competency for VTE, it

may not be satisfactory to determine the importance levels by comparing only one of the factors with each other because such an approach is limited only to a calculation based on comparison within the group of factors. However, the importance or weight of VTE factors may differ in terms of internal interactions as well as external criteria. As a matter of fact, the importance levels of the factors affecting the competency of VTE may vary according to the criteria of VTE. For instance, it is not possible to say that the VTE competency factors of a developed economy and the VTE competency factors and objectives of a less developed economy may be the same. Moreover, the objectives of a VTE institution within the same economy or sector may differ. These situations indicate that the academic unit providing VTE may have various objectives. For example, vocational schools of higher education or faculties may have various objectives such as gaining academic competence, job placement, competing and branding. Therefore, the competency of VTE needs to be evaluated according to the objectives of the VSHE or faculty. This necessitates the consideration of the differences in objectives in determining the competency of VTE. Another point is that the importance of the objectives of the VSHE or faculty as an educational institution may differ. In this case, the relative importance of the objectives of the educational institution should be considered in determining the competency of VTE. Another point that should be mentioned here is that when considered as content, one of the objectives is often not independent of one another. In other words, the interactions of objectives may be possible. For instance, it is unthinkable that the quality education objective of a



school offering VTE is independent of the job placement or branding objective. Therefore, an approach that takes into account the interaction and relationship of the objectives should be used in the evaluation of the VTE competency issue. Another issue that needs to be expressed is the evaluation of the competency of VTE in a holistic approach relying on the priorities determined in the context of the objectives of the school.

No studies were encountered in the literature examining the competency of a VSHE or faculty providing VTE by considering the above-mentioned issues. This study intends to suggest a model for evaluating the competency of a VSHE which provides education and training in the field of VTE with a holistic approach.

### **Method**

This study utilized ANP and TOPSIS techniques in the development of the suggested model for measuring and evaluating the competency of VTE. Analytic network process (ANP), which is one of the multiple-criteria decision-making techniques, is a technique allowing the calculation of the degree of possible relationships between the criteria contained in multidimensional problems. ANP was developed mainly on the basics of analytic hierarchy process (AHP). ANP was brought to the literature as a result of the pioneering work of Saaty (Saaty and Takizawa, 1986; Saaty, 1996; Saaty, 1999). ANP allows the calculation of priorities of decision problems with one-way non-hierarchical relationships. In other words, it can make a prioritization considering the dependencies between the criteria of the decision

problem. The main difference between AHP and ANP is that ANP is able to calculate composite weights by means of supermatrix which was developed by considering the interrelationships between factors or levels in the model (Shyur, 2006). Supermatrix is a partitioned matrix. Each sub-matrix shows the relationships between two components or clusters in a network structure (Shyur, 2006). Saaty (1996) described the concept of supermatrix in a similar way to the Markov chain process. In this study, the formation of supermatrix was created according to the concept of Saaty and Takizawa (1986). This is because, as Shyur (2006) stated, it has an easier processing. In the relevant literature, it was seen that the procedures for the ANP was explained in various manners and steps (Saaty, 1996; Chung, Lee, and Pearn, 2005; Shyur, 2006; Cheng and Li, 2007; Lee, Kim, Cho and Park, 2009). In this study, the steps of ANP procedures explained by Shyur (2006) were used. These steps are as follows:

**Step 1: Binary comparisons of criteria in the context of non-dependence.** Decision makers make a pairwise comparison of the criteria in the model. In this study, the competency objectives of VTE were expressed as criteria in ANP model. The expert group answered the following question in comparing the criteria: “Which objective is more important in terms of VTE competency and how important is it? The answers of the expert group were given according to the Saaty’s 1-9 (Saaty, 1980) scale (Table 1). Each pair of criteria is judged once. The reciprocal value is the opposite of the comparison. When the binary comparisons are completed, the local weights ( $w_1$ ) vector are found with the  $Aw_1 = \lambda_{max}w_1$  equation.

The term  $\lambda_{\max}$  in the equation denotes the largest eigenvalue of the binary comparison matrix **A**. The resulting vector is normalized. For this, each value is divided by the column total to symbolize the standardized local weight vector  $w_2$  (Shyur, 2006).

*Table 1. Levels of Importance in Binary Comparisons*

$a_{ij}$	Definition
1	Equal importance
3	Weak importance
5	Strong importance
7	Very strong importance
9	Extreme importance
2,4,6,8	Intermediate values

In ANP, it is expected that the pairwise comparisons of the criteria constituting the problem are consistent. Therefore, consistency ratios of binary comparison matrices are calculated. The consistency calculation is determined by calculating the inconsistency of the binary comparison matrix. Primarily, the consistency index (C.I.) is calculated to find out the consistency ratio (C.R.). Consistency index is acquired through  $C.I. = (\lambda_{\max} - n) / (n - 1)$  equation. The term “n” in the equation indicates the size of the square matrix. Upon calculating the consistency index, the inconsistency index is calculated through  $(C.R.) = (C.I.) / (R.I.)$  equation. The size of the comparison matrix determines the random consistency index (R.I.) value in the equation (Kim, Jang, and Lee, 2013; Deng, Hu, Deng, Mahadevan, 2014). The consistency ratios of the comparison matrices vary according to the size of the matrix (Konstantinos, Georgios, and Garyfalos, 2019) however, if the

inconsistency rate is less than 0.10 when the size of the matrix is  $n \geq 5$ , the matrix is consistent. Otherwise, if the binary comparisons are found to be inconsistent, the binary comparisons are repeated (Dweiri, Kumar, Khan, and Jain, 2016).

**Step 2: Determination of interdependence between criteria.** The expert group examines the impact of each criterion on the other. This is done through binary comparison. The question asked in the binary comparison for this study was: Which VTE competency objective affects the objective of other VTE competency included in the model? And to what extent does it affect? Decision makers answer this question. Various binary comparison matrices are created for each VTE competency objective. Binary comparison matrices are needed to determine the relative effects of dependent relationships of criteria. For these matrices, normalized principal eigenvectors are calculated. In the interdependence weight matrix of criteria B, where zeros are appointed to the eigenvector weights of the criteria from which a given criterion is given, the calculated eigenvectors are shown as a column element.

**Step 3: Calculation of dependence priorities of criteria.** In this step, the dependence priorities of the criteria are calculated by synthesizing the results of the previous two steps:  $w_c = Bw_2^T$ .

Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) was the second multiple-criteria decision-making technique employed in this study. TOPSIS was developed by Hwang and Yoon (1981). The main feature of the TOPSIS technique is its ability to

rank and prioritize multiple alternatives according to multiple criteria. In TOPSIS method, the optimal alternative is determined as closest to positive ideal solution and farthest away from negative ideal solution (Walczak and Rutkowska, 2017). In the related literature, it is seen that TOPSIS technique was used in various multiple-criteria issues (Dağdeviren, Yavuz and Kılınc, 2009; Ding and Zeng, 2015; Karahalios, 2017; Meng, Shao, and Zhu, 2018; Pelegrina, Duarte, and Romanoa, 2019; Konstantinos, Georgios, and Garyfalos, 2019; Ramya and Devadas, 2019). Shyur (2006) The procedures for the TOPSIS technique are demonstrated below:

*Step 1: Formation of decision matrix. The structure of the decision matrix is as follows:*

$$D = \begin{matrix} & F_1 & F_2 & \dots & F_j & \dots & F_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_i \\ \vdots \\ A_j \end{matrix} & \left[ \begin{matrix} f_{11} & f_{12} & \dots & f_{1j} & \dots & f_{1n} \\ f_{21} & f_{22} & \dots & f_{2j} & \dots & f_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{i1} & f_{i2} & \dots & f_{ij} & \dots & f_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ f_{m1} & f_{m2} & \dots & f_{mj} & \dots & f_{mn} \end{matrix} \right. \end{matrix}$$

$A_j$  in the matrix denotes the alternatives to the problem. With  $i = 1, m$ ; and  $F_j$  terms show the properties and criteria,  $j = 1, \dots, n$ . The term  $f_{ij}$  is a crisp value. This value is determined by the comparison of each alternative  $A_i$  and each criterion  $F_j$ .

**Step 2:** Calculation of the standardized decision matrix. The normalized  $R(= [r_{ij}])$  value is computed utilizing the following equation:

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{j=1}^n f_{ij}^2}}, j = 1, \dots, n;$$

$i = 1, \dots, m.$

**Step 3:** Calculation of the weighted normalized decision matrix. This is computed by multiplying the weights of the normalized decision matrix. The weighted normalized values ( $v_{ij}$ ) are computed using:  $v_{ij}=w_j r_{ij}, j = 1, \dots, n; i = 1, \dots, m, w_j.$  The weight of  $j.$  property or criterion is demonstrated through this equation.

**Step 4:** Identification of ideal and negative ideal solutions. The benefit criterion is displayed by  $J$  and the cost criterion is shown by  $J'$ , which follows as:

$$V^+ = \{v_1^+, \dots, v_n^+\} = \{(\max_i v_{ij} | j \in J), (\min_i v_{ij} | j \in J')\}$$

$$V^- = \{v_1^-, \dots, v_n^-\} = \{(\min_i v_{ij} | j \in J), (\max_i v_{ij} | j \in J')\},$$

**Step 5:** Calculation of separation measurements through  $m$ -dimensioned Euclidean distance. Separation for each alternative from the ideal solution ( $D_i^+$ ) is done as follows:

$$D_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}, i = 1, \dots, m,$$

Likewise, separation of each alternative from the negative ideal ( $D_i^-$ ) is calculated as follows:

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, i = 1, \dots, m$$

**Step 6:** Calculation of the relative distance to the ideal solution and identification of order of choices.

$$C_i = \frac{D_i^-}{D_i^+ + D_i^-}, i = 1, \dots, m. C_i \text{ A value between 0}$$

and 1 is assigned to the index value. Better performance alternatives are demonstrated by indexes with large values.

In the light of the information above, the steps of the proposed model in this study are as follows:

*Step 1.* Formation of expert team.

*Step 2.* Determination of competency factors of VTE.

*Step 3.* Describing the objectives of the VSHE.

*Step 4.* Establishing a binary comparison matrix to determine the weights of the objectives of the VSHE.

*Step 5.* Ranking the VTE competency factors according to the objectives and calculating their weights through TOPSIS technique.

*Step 6.* Calculation of the competency level of VTE.

*Step 7.* Determination of deviations from the competency level of VTE.

## Results

In this study, the results of the model for measuring and evaluating the competency of VTE were presented in this section in accordance with the steps given in the method section. Accordingly, the results of the study were given in six steps.

**Step 1. Formation of the expert team.** In this step of the study, a team was formed that provided the data and opinions required for the research. In this study, the expert group consisted of an instructor working in the VSHE, the researcher of the study and an experienced instructor working in a VSHE.

**Step 2. Determination of competency factors of VTE.** In this step, VTE competency factors in the model of the study were determined. Firstly, the studies on the subject of quality of VTE studies in the literature (Sahu, Shrivastava, and Shrivastava, 2013; Eskandaripour, Hajhosseinnezdah, Aliasgari, and Hosseinihah, 2019) were examined. The factors identified in the studies in the relevant literature were selected by the expert team based on the opinions of the director at the VSHE where the research was conducted. As a result of these procedures, 43 factors determined for the VSHE within the scope of the research are as follows:

- Communication skills of instructors
- Instructors' effectiveness for student participation and collaboration
- Instructors' use of nonverbal skills
- Instructors' presentation skills

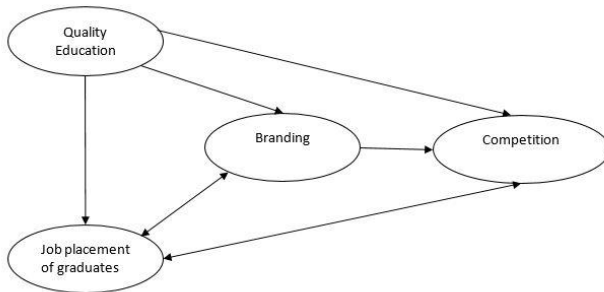


- Instructors' good command of course subjects
- Instructors' ability to explain, interpret and examine
- Instructors' ability to use examples
- Instructors' ability to encourage student learning
- Instructors' ability to use appropriate teaching strategies
- Quality of teaching method of instructors
- Classroom management skills of instructors
- Instructors' maintenance of classroom order
- Starting and finishing courses on time by instructors
- Checking students' attendance
- Determining the readiness of students for vocational education
- Optimal use of time
- Educational evaluation skills of instructors
- Quality of exams in terms of validity, reliability and feedback
- Consistency of course content with exam methods
- Educational material
- Information and communication service
- Library reference resources, journals, etc.
- Internet and original databases
- Physical organization of classroom environments
- Culture of faculty / VSHE

- Consultancy services
- Instructors' job satisfaction
- In-service training of non-academic staff
- Area of faculty/VSHE
- Classroom capacities
- Competency of workshops
- Competency of laboratories
- Quality of laboratory materials
- Quality of health facilities
- Cultural and sports facilities
- Evaluation process of teaching quality
- Collaboration and networking with institutions
- Exchange programs
- Curriculum design and revision
- Academic planning and monitoring
- Continuous assessment and monitoring
- Convenient working environment
- Accommodation and nutrition

***Step 3. Describing the objectives of the VSHE.*** In this step of the study, the objectives of the VSHE providing technical education were determined. For this purpose, as a result of the interviews conducted with the managers of the VSHE where the study was conducted, quality education, job placement of graduates, competition and

branding were determined by the expert group as the objectives of VSHE providing VSHE. Moreover, as emphasized in the problem stated in the introduction part of the study, it is not possible to think that one of the objectives is independent of another. There are possible situations of interaction of objectives. In this study, the interaction of the objectives of the VSHE within the scope of the research was determined by the expert group of the study. Accordingly, the interaction between the objectives determined was given in Figure 1. According to the pattern in Figure 1, it is seen that quality education affects job placement of graduates, branding and competition objectives. The objective of job placement of graduates affects the objective of branding and competition. On the other hand, it is seen that the objective of branding affects the objectives of job placement of graduates and competition. The latest situation in the pattern affects the objective of competition and the job placement of graduates.



***Step 4. Establishing a binary comparison matrix to determine the weights of the objectives of the VSHE.***

Due to the requirements of ANP's mathematical process

(Saaty, 1996; Chung, Lee, Pearn, 2005; Shyur, 2006; Cheng and Li, 2007; Lee, Kim, Cho, and Park, 2009), primarily, the importance of the objectives of the VSHE was calculated for the situation where the factors are independent of one another. For this purpose, primarily, binary comparisons of the objectives were made relying on the expert opinion. Saaty’s (1980) scale was used in binary comparisons. Table 2 shows the pairwise comparisons of the objectives and the consistency ratio with the calculated local weights.

*Table 2: Pairwise comparisons of the objectives in terms of independence*

Objectives	QE	JP	BD	CP	Local weights	CR
Quality Education (QE)	1	2	3	3	0.4412	0.05
Job Placement of Graduates (JP)		1	3	2	0.2898	
Branding (BD)			1	2	0.1520	
Competition (CP)				1	0.1170	

Another process performed in the fourth step was to determine the internal dependence matrix. For this purpose, binary comparisons were made based on the relationships between the objectives in Figure 1. Binary comparisons based on internal dependence were arranged in Tables 3-5. Table 3 shows the pairwise comparisons based on job placement criteria and their local weights and consistency ratios. Pairwise comparisons made according to branding criteria were presented in Table 4 and pairwise comparison of objectives that affect competition was presented in Table 5.

*Table 3: Factors affecting job placement of graduates*

Objectives	QE	BD	CP	Local weights	CR
Quality education (QE)	1	3	3	0.594	0.05
Branding (BD)		1	2	0.249	
Competition (CP)			1	0.157	

*Table 4: Factors affecting branding*

Objectives	QE	JP	Local weights
Quality education (QE)	1	3	0.750
Job placement of graduates (JP)		1	0.250

*Table 5: Factors affecting competition*

Objectives	QE	JP	BD	Local weights	CR
Quality education (QE)	1	2	3	0.528	0.05
Job placement of graduates (JP)		1	3	0.333	
Branding (BD)			1	0.140	

The weights calculated in Table 2-5 above and the dependent weights of the objectives according to the pattern in Figure 1 were given in Table 6. As can be seen in Table 6, dependent weights were calculated by multiplying the weights calculated according to the interaction of the objectives and the weights calculated according to the assumption of non-relationship between the objectives.

Table 6. Dependent Weights Matrix for Main Factors

	QE	JP	BD	CP		Non-relational weights	Dependent weights		
W=	QE	0.250	0.594	0.750	0.527	X	0.441	=	0.4582
	JP	0.250	0.000	0.250	0.332		0.290		0.1871
	BD	0.250	0.249	0.000	0.140		0.152		0.1988
	CP	0.250	0.157	0.000	0.000		0.117		0.1558

**Step**

**5. Ranking the VTE competency factors according to the objectives and calculating their weights through TOPSIS technique.** In this step of the study, the ranking of the VTE competency factors and their weights were calculated by TOPSIS technique. The analysis of the TOPSIS technique was based on the objectives of the VSHE. The quality factors of VTE included in the analysis were determined in the second step of the study. The order of importance determined according to the values of TOPSIS analysis was given in the last column of Table 7. The relative weights ( $sdzC_i$ ) were calculated by standardizing the values determined by TOPSIS analysis. The second column of Table 9 shows the standardized  $C_i$  values.

*Table 7: Priorities of VTE Quality Factors According to Objectives of VSHE*

Communication skills of instructors	5	2	3	4	0.03863	0.07833	0.66975	8
Instructors' effectiveness for student participation and collaboration	5	2	4	3	0.03778	0.07874	0.67578	6
Instructors' use of nonverbal skills	4	2	3	3	0.04441	0.05905	0.57075	19
Instructors' presentation skills	5	2	3	3	0.04014	0.07757	0.65899	9
Instructors' good command of course subjects	5	2	4	3	0.03778	0.07874	0.67578	7
Instructors' ability to explain, interpret and examine	4	2	2	3	0.04774	0.05853	0.55079	21
Instructors' ability to use examples	5	2	3	2	0.04254	0.07731	0.64505	10
Instructors' ability to encourage student learning	5	4	4	3	0.01882	0.08529	0.81920	1
Instructors' ability to use appropriate teaching strategies	4	3	3	2	0.03874	0.06205	0.61565	13
Quality of teaching method of instructors	5	3	4	4	0.02525	0.08199	0.76455	4
Classroom management skills of instructors	4	2	2	2	0.04977	0.05819	0.53898	23
Instructors' maintenance of classroom order	3	2	2	2	0.05968	0.03974	0.39974	34
Starting and finishing courses on time by instructors	3	3	3	3	0.04885	0.04564	0.48303	27
Checking students' attendance	3	1	2	3	0.06559	0.03854	0.37011	37
Determining the readiness of students for vocational education	4	3	2	3	0.04010	0.06187	0.60675	16
Optimal use of time	3	2	3	3	0.05529	0.04099	0.42578	30
Educational evaluation skills of instructors	4	2	3	3	0.04441	0.05905	0.57075	20
Quality of exams in terms of validity, reliability and feedback	4	2	4	3	0.04229	0.06059	0.58892	18
Consistency of course content with exam methods	4	2	5	4	0.04010	0.06401	0.61480	14

Educational material	4	1	4	4	0.05107	0.06047	0.54211	22
Information and communication service	1	4	5	5	0.07691	0.04600	0.37425	36
Library reference resources, journals, etc.	4	2	5	5	0.03960	0.06554	0.62333	12
Internet and original databases	3	2	4	4	0.05247	0.04454	0.45909	28
Physical organization of classroom environments	3	3	4	4	0.04564	0.04885	0.51697	24
Culture of faculty / VSHE	3	2	3	3	0.05529	0.04099	0.42578	31
Consultancy services	3	4	5	5	0.03974	0.05968	0.60026	17
Instructors' job satisfaction	2	1	3	3	0.07617	0.02150	0.22014	43
In-service training of non-academic staff	2	2	3	3	0.06974	0.02442	0.25938	42
Area of faculty/VSHE	2	1	4	4	0.07416	0.02767	0.27170	40
Classroom capacities	3	1	3	3	0.06321	0.03932	0.38353	35
Adequacy of workshops	5	3	5	5	0.02316	0.08501	0.78588	2
Adequacy of laboratories	5	3	5	5	0.02316	0.08501	0.78588	3
Quality of laboratory materials	3	2	3	3	0.05529	0.04099	0.42578	32
Quality of health facilities	2	2	4	4	0.06753	0.02999	0.30754	39
Cultural and sports facilities	2	1	4	4	0.07416	0.02767	0.27170	41
Evaluation process of teaching quality	3	2	5	5	0.05150	0.04989	0.49202	26
Collaboration and networking with institutions	2	5	5	5	0.05703	0.05845	0.50616	25
Exchange programs	3	5	5	5	0.03802	0.06709	0.63828	11
Curriculum design and revision	5	2	5	5	0.03474	0.08261	0.70395	5
Academic planning and monitoring	4	2	5	4	0.04010	0.06401	0.61480	15
Continuous assessment and monitoring	3	2	4	4	0.05247	0.04454	0.45909	29
Convenient working environment	3	1	4	4	0.06077	0.04300	0.41443	33
Accommodation and nutrition	2	1	5	5	0.07347	0.03564	0.32665	38



**Step 6. Calculation of competency level of VTE.** In this step of the study, the VTE competency level of the VSHE within the scope of the research was calculated. An instructor working in the VSHE subject to the study evaluated the current status of each VTE competency factor with the scale provided in Table 8 (Yüksel and Dağdeviren 2006). “For example, what is the favorable working environment?” was the evaluation question. This question was responded by one of the quality levels given in Table 8. For example, the answer given to this question was Average (AR) and 0.6 was the corresponding value.

Table 8. VTE Competency Evaluation Scale

Level	Value
Sufficient (SF)	1.0
Good (GD)	0.8
Average (AR)	0.6
Insufficient (IS)	0.4
Very insufficient (VIS)	0.2
Not available(NA)	0.0

The answers given to the questions of an instructor working in the VSHE within the scope of the research were presented in the third column of Table 9. The numerical values corresponding to the answers are in the fourth column of Table 9. In the last column of Table 9, the levels of the VTE competency factors of the VSHE within the scope of the research were given. The sufficiency level (CL) in the last column of Table 9 was calculated by multiplying the standardized values ( $sdzC_i$ ) of the factors and the actual state (AS). In the last line

of Table 9, the total competency level of VTE (TCL) calculated for the VSHE within the scope of the research was given. This value can be between 0 and 1 ( $0 \leq \text{TCL} \leq 1$ ). The total competency level of VTE calculated for this study was found to be 0.6642.

Table 9. Calculation of VTE Competency Level

VTE Evaluation Factors	Standardized C <sub>i</sub> Values (sdzC <sub>i</sub> )	Actual state (AS)	Scale Value (VAS) (Table 5)	Competency Level (CL)=[(sdzC <sub>i</sub> )X (VAS)]
Communication skills of instructors	0.02956	GD	0.8	0.0237
Instructors' effectiveness for student participation and collaboration	0.02983	GD	0.8	0.0239
Instructors' use of nonverbal skills	0.02519	GD	0.8	0.0202
Instructors' presentation skills	0.02909	GD	0.8	0.0233
Instructors' good command of course subjects	0.02983	SF	1	0.0298
Instructors' ability to explain, interpret and examine	0.02431	GD	0.8	0.0195
Instructors' ability to use examples	0.02847	SF	1	0.0285
Instructors' ability to encourage student learning	0.03616	AR	0.6	0.0217
Instructors' ability to use appropriate teaching strategies	0.02718	AR	0.6	0.0163
Quality of teaching method of instructors	0.03375	GD	0.8	0.0270
Classroom management skills of instructors	0.02379	AR	0.6	0.0143
Instructors' maintenance of classroom	0.01765	GD	0.8	0.0141
Starting and finishing courses on time by instructors	0.02132	AR	0.6	0.0128
Checking students' attendance	0.01634	IS	0.4	0.0065
Determining the readiness of students for vocational education	0.02678	AR	0.6	0.0161
Optimal use of time	0.01880	AR	0.6	0.0113
Educational evaluation skills of instructors	0.02519	GD	0.8	0.0202
Quality of exams in terms of validity, reliability and feedback	0.02600	GD	0.8	0.0208
Consistency of course content with exam	0.02714	GD	0.8	0.0217
Educational material	0.02393	AR	0.6	0.0144
Information and communication service	0.01652	AR	0.6	0.0099
Library reference resources, journals, etc.	0.02752	GD	0.8	0.0220
Internet and original databases	0.02027	SF	1	0.0203
Physical organization of classroom environments	0.02282	SF	1	0.0000
Culture of faculty / VSHE	0.01880	AR	0.6	0.0188

Consultancy services	0.02650	AR	0.6	0.0159
Instructors' job satisfaction	0.00972	IS	0.4	0.0058
In-service training of non-academic staff	0.01145	IS	0.4	0.0046
Area of faculty/VSHE	0.01199	SF	1	0.0048
Classroom capacities	0.01693	AR	0.6	0.0169
Adequacy of workshops	0.03469	GD	0.8	0.0208
Adequacy of laboratories	0.03469	GD	0.8	0.0278
Quality of laboratory materials	0.01880	GD	0.8	0.0150
Quality of health facilities	0.01358	IS	0.4	0.0109
Cultural and sports facilities	0.01199	IS	0.4	0.0048
Evaluation process of teaching quality	0.02172	AR	0.6	0.0087
Collaboration and networking with institutions	0.02234	AR	0.6	0.0134
Exchange programs	0.02818	VIS	0.2	0.0169
Curriculum design and revision	0.03107	IS	0.4	0.0062
Academic planning and monitoring	0.02714	IS	0.4	0.0109
Continuous assessment and monitoring	0.02027	IS	0.4	0.0081
Convenient working environment	0.01829	AR	0.6	0.0073
Accommodation and nutrition	0.01442	AR	0.6	0.0087
	1.00000	Total Competency Level (TCL)		0.6642

**Step 7. Determination of deviations from the competency level of VTE.** In this step of the study, deviations from the VTE competency levels calculated for the VSHE evaluated according to the proposed model were determined. Thus, the competency factors and the level of deficiency of the VSHE within the scope of the research were determined (Table 10). Detailed results of the competency factors of VTE in Table 10. Factors were given in the first column, the calculated relative weights of VTE factors in the second column, competency levels of the factors of VTE of the VSHE within the scope of the study in the third column, and the fourth column displays the deviation values. The difference between weight value and competency level is defined as the deviation value.

Table 10. Detailed Evaluation of VTE Competency Factors

Factors	Standardized C <sub>v</sub> Values (SdzC <sub>v</sub> )	Competency Levels (CL)=[(sdzC <sub>v</sub> )X (VAS)]	Deviation (SdzC <sub>v</sub> )-(CL)
Communication skills of instructors	0.02956	0.0237	0.00591
Instructors' effectiveness for student participation and collaboration	0.02983	0.0239	0.00597
Instructors' use of nonverbal skills	0.02519	0.0202	0.00504
Instructors' presentation skills	0.02909	0.0233	0.00582
Instructors' good command of course subjects	0.02983	0.0298	0.00000
Instructors' ability to explain, interpret and examine	0.02431	0.0195	0.00486
Instructors' ability to use examples	0.02847	0.0285	0.00000
Instructors' ability to encourage student learning	0.03616	0.0217	0.01446
Instructors' ability to use appropriate teaching strategies	0.02718	0.0163	0.01087
Quality of teaching method of instructors	0.03375	0.0270	0.00675
Classroom management skills of instructors	0.02379	0.0143	0.00952
Instructors' maintenance of classroom order	0.01765	0.0141	0.00353
Starting and finishing courses on time by instructors	0.02132	0.0128	0.00853
Checking students' attendance	0.01634	0.0065	0.00980
Determining the readiness of students for vocational education	0.02678	0.0161	0.01071
Optimal use of time	0.01880	0.0113	0.00752
Educational evaluation skills of instructors	0.02519	0.0202	0.00504
Quality of exams in terms of validity, reliability and feedback	0.02600	0.0208	0.00520
Consistency of course content with exam methods	0.02714	0.0217	0.00543
Educational material	0.02393	0.0144	0.00957
Information and communication service	0.01652	0.0099	0.00661
Library reference resources, journals, etc.	0.02752	0.0220	0.00550
Internet and original databases	0.02027	0.0203	0.00000
Physical organization of classroom environments	0.02282	0.0000	0.02282
Culture of faculty / VSHE	0.01880	0.0188	0.00000
Consultancy services	0.02650	0.0159	0.01060
Instructors' job satisfaction	0.00972	0.0058	0.00389
In-service training of non-academic staff	0.01145	0.0046	0.00687
Area of faculty/VSHE	0.01199	0.0048	0.00720
Classroom capacities	0.01693	0.0169	0.00000
Adequacy of workshops	0.03469	0.0208	0.01388
Adequacy of laboratories	0.03469	0.0278	0.00694
Quality of laboratory materials	0.01880	0.0150	0.00376
Quality of health facilities	0.01358	0.0109	0.00272
Cultural and sports facilities	0.01199	0.0048	0.00720
Evaluation process of teaching quality	0.02172	0.0087	0.01303
Collaboration and networking with institutions	0.02234	0.0134	0.00894
Exchange programs	0.02818	0.0169	0.01127
Curriculum design and revision	0.03107	0.0062	0.02486
Academic planning and monitoring	0.02714	0.0109	0.01628
Continuous assessment and monitoring	0.02027	0.0081	0.01216
Convenient working environment	0.01829	0.0073	0.01098

### Conclusion

In this study, a model for measuring and evaluating the competency of VTE was proposed. The proposed model was structured with ANP and TOPSIS techniques which are multiple-criteria decision-making techniques. This is mainly due to the multi-dimensional and multiple-

criteria nature of the evaluation of VTE competency. As a matter of fact, studies in the related literature (Tuncer and Tanaş, 2019; Eskandaripour, Hajihosseinnezdah, Alinasgari, and Hosseinkhah, 2019) showed that competency of VTE is a component of many factors. Therefore, it is necessary to evaluate the competency of VTE with the models and techniques appropriate to this feature in terms of reflecting the real situation.

When the results of this study were examined, it was found that the proposed ANP-TOPSIS-based model was suitable for measuring and evaluating the VTE competency. The model proposed in the study has a feature considering the objectives of the VSHE providing VTE. Additionally, the possible interaction between the objectives was considered in the proposed model in the evaluation of the competency of VTE. These two characteristics of the model reinforce the validity of the assessment of the competency of the vocational and technical VSHE since the importance level of each factor of VTE competency can vary according to the objectives of the VSHE. For this reason, it is important to carry out measurement and evaluation by considering the importance levels of VTE competency factors on the basis of objectives. In this study, this issue was considered in the proposed model and the importance of this issue was seen in the results. Consideration of interaction or dependence between objectives is the second factor that increases the validity of the results because it is not possible to think of the objectives independently of each other. One objective may have an effect on the other objective or objectives. The model

proposed in this study considered this issue with ANP technique. Another feature of the study is that it enables to evaluate the competency of VTE in detail. With the suggested model in the study, the level of competency of each VTE factor can be answered. In addition, the level of competency of the proposed model VTE institution as a whole could be determined.

It can be said that the VTE competency evaluation model proposed in the study is applicable in vocational schools. However, the proposed model can be modified and applied in VSHEs that differ in content and number of objectives or factors. It can be said that the model proposed in the study can be used for other types of education. Evaluation can be done by taking the competency factors to be determined for other types of education instead of VTE factors included in the proposed model. Similarly, in the model to be modified, the objectives may vary in number or content. As a result, different types of education can be evaluated by differentiating the relationships between the factors, objectives and objectives that are included in the model to be modified.

In the continuation of this study, further studies may allow to determine the drawbacks of the proposed model or make it suitable for a more valid measurement. In the first of these studies, the relationship between objectives could be determined through DEMATEL technique. Another study that can be done in the future may use fuzzy numbers instead of crisp numbers used in the measurement process. The use of fuzzy numbers may provide a more specific measure of the evaluation of qualitative factors.

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**SOME FUNCTORIAL  
RELATIONS OF TWO-  
CROSSED MODULES  
ON COMMUTATIVE  
ALGEBRAS**

**ÖLÜM  
9**

**Ummahan Ege Arslan<sup>1</sup>**

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## Introduction

Crossed modules of groups were initially defined by Whitehead [13, 14] as models for

(homotopy) 2-types. The commutative algebra case of crossed modules is contained in the

paper of Lichtenbaum and Schlessinger [7] and also the work of Gerstenhaber [4] under

different names. Some categorical results and Koszul complex link are also given by Porter

[9, 10]. Conduché, [3], in 1984 described the notion of 2-crossed modules as a model

for 3-types. The commutative algebra version of 2-crossed modules has been defined by

Grandjeán and Vale [5]. Arvasi [2] has study related with that construction.

In this study some examples of crossed modules and 2-crossed modules are given. Some

functorial relations among the category  $k\text{-Alg}$  of  $k$ -algebras,  $X\text{Mod}$  and  $X_2\text{Mod}$  are researched and it is obtained an adjunction relation between  $X_2\text{Mod}$  and  $k\text{-Alg}$  as a functorial

result.

## Conventions

Throughout this paper  $k$  will be a fixed commutative ring and  $R$  will be a  $k$ -algebra with identity. All algebras will be commutative.



## Crossed Modules and Two-Crossed Modules of Algebras

Crossed modules of groups were initially defined by Whitehead [13, 14] as models for

(homotopy) 2-types. Conduché, [3], in 1984 described the notion of 2-crossed module as

a model for 3-types. Both crossed modules and 2-crossed modules have been adapted for

use in the context of commutative algebras in [5, 10].

A crossed module is an algebra morphism  $\partial: C \rightarrow R$  with action of  $R$  on  $C$  satisfying

$$\partial(r \cdot c) = r \partial(c) \quad \text{and} \quad \partial(c) \cdot c' = cc'$$

for all  $c, c' \in C, r \in R$ . When the first equation is satisfied,  $\partial$  is called pre-crossed module.

If  $(C, R, \partial)$  and  $(C', R', \partial')$  are crossed modules, a morphism,

$$(\theta, \phi): (C, R, \partial) \rightarrow (C', R', \partial'),$$

of crossed modules consists of  $k$ -algebra homomorphisms  $\theta: C \rightarrow C'$  and  $\phi: R \rightarrow R'$

such that

$$(i) \partial' \theta = \phi \partial \quad (ii) \theta(r \cdot c) = \phi(r) \cdot \theta(c)$$

for all  $r \in R, c \in C$ . We thus get the category  $XMod$  of crossed modules.

Examples of crossed modules

1. An inclusion map  $i: I \rightarrow R$  is a crossed module where  $I$  is an ideal of  $R$ . Conversely,

given any crossed module,  $\partial: C \rightarrow R$  we get that  $\partial(C)$  is an ideal of  $R$ .

2. Any  $R$ -module  $M$  can be considered as an  $R$ -algebra with zero multiplication and

hence the zero morphism  $0: M \rightarrow R$  is a crossed module. Conversely, if  $(C, R, \partial)$  is a

crossed module,  $\text{Ker } \partial$  is an ideal in  $C$  and  $\text{Ker } \partial$  has a  $R/\partial(C)$ -module structure, since

$\text{Ker } \partial$ , inherits a natural  $R$ -module structure and  $\partial(C)$  acts trivially on  $\text{Ker } \partial$ . Moreover,

$\partial(C)$  acts trivially on  $\text{Ker } \partial$ , hence  $\text{Ker } \partial$  has a natural  $R/\partial(C)$ -module structure.

3. Let  $M(R)$  be multiplication algebra defined by Mac Lane [6] (see also [8]) as the

set of all multipliers  $\delta: R \rightarrow R$  such that for all  $r, r' \in R$ ,  $\delta(r r') = r \delta(r')$  where  $R$  is a

commutative  $k$ -algebra and  $\text{Ann}(R) = 0$  or  $R^2 = R$ . Then  $\mu: R \rightarrow M(R)$  is a crossed module

given by  $\mu(r) = \delta r$  with  $\delta(r r') = r r'$  for all  $r, r' \in R$ . (See [1] for details).

4. Any epimorphism of algebras  $C \rightarrow R$  with the kernel in the annihilator of  $C$  is a

crossed module, with  $r \in R$  acting on  $c \in C$  by  $r \cdot c = \bar{c}c$ , where  $\bar{c}$  is any element in the

pre-image of  $r$ .

We see that crossed modules provide a simultaneous generalization of the concepts of

ideals and modules. Also any algebra together with its multiplication algebra gives rise to

a crossed module and any algebra may itself be regarded as a crossed module.

Grandjeán and Vale [5] have given a definition of 2-crossed modules of algebras. The

following is an equivalent formulation of that concept.

A *2-crossed module* of  $k$ -algebras consists of a complex of  $P$ -algebras  $L \xrightarrow{\partial_2} M \xrightarrow{\partial_1} P$

together with an action of  $P$  on all three algebras and a  $P$ -linear mapping

$$\{-, -\} : M \times M \rightarrow L$$

which is often called the Peiffer lifting such that the action of  $P$  on itself is by multiplication,

$\partial_1$  and  $\partial_2$  are  $P$ -equivariant,

$$\mathbf{PL1} : \partial_2\{m_0, m_1\} = m_0m_1 - \partial_1(m_1) \cdot m_0$$

$$\mathbf{PL2} : \{\partial_2(l_0), \partial_2(l_1)\} = l_0l_1$$

$$\mathbf{PL3} : \{m_0, m_1m_2\} = \{m_0m_1, m_2\} + \partial_1(m_2) \cdot \{m_0, m_1\}$$

$$\mathbf{PL4} : \{m, \partial_2(l)\} = m \cdot l \text{ and } \{\partial_2(l), m\} = m \cdot l - \partial_1(m) \cdot l$$

$$\mathbf{PL5} : p \cdot \{m_0, m_1\} = \{p \cdot m_0, m_1\} = \{m_0, p \cdot m_1\}$$

for all  $m, m_0, m_1, m_2 \in M, l, l_0, l_1 \in L$  and  $p \in P$ .

Note that we have not specified that  $M$  acts on  $L$ . We could have done that as follows:

if  $m \in M$  and  $l \in L$ , define

$$m \cdot l = \{m, \partial_2(l)\}.$$

Since  $\partial_2(m \cdot l) = \partial_2(\{m, \partial_2(l)\}) = m\partial_2(l) - \partial_1(\partial_2(l)) \cdot m = m\partial_2(l) - 0 \cdot m = m\partial_2(l)$ , and  $\partial_2(l) \cdot l' = \{\partial_2(l), \partial_2(l')\} = ll'$ ,  $(L, M, \partial_2)$  becomes a crossed module. We denote such a 2-crossed module of algebras by  $\{L, M, P, \partial_2, \partial_1\}$ .

A morphism of 2-crossed modules is given by the following diagram

$$\begin{array}{ccccc} L & \xrightarrow{\partial_2} & M & \xrightarrow{\partial_1} & P \\ f_2 \downarrow & & f_1 \downarrow & & f_0 \downarrow \\ L' & \xrightarrow{\partial'_2} & M' & \xrightarrow{\partial'_1} & P' \end{array}$$

where

$$\begin{aligned} f_0\partial_1 &= \partial'_1f_1, f_1\partial_2 = \partial'_2f_2 \\ f_1(p \cdot m) &= f_0(p) \cdot f_1(m), f_2(p \cdot l) \\ &= f_0(p) \cdot f_2(l) \end{aligned}$$

for all  $m \in M, l \in L, p \in P$  and

$$\{-, -\} f_1 \times f_1 = f_2 \{-, -\}.$$

We thus get the category of 2-crossed modules denoting it by  $X_2\text{Mod}$  and when the

morphism  $f_0$  above is the identity we will get  $X_2\text{Mod}/P$  the category of 2-crossed modules

over fixed algebra  $P$ .

If we have a 2-crossed module  $L \xrightarrow{\partial_2} M \xrightarrow{\partial_1} P$  with the Peiffer lifting  $\{m, m'\} = 0$  then

- i)  $(M, P, \partial_1)$  becomes a crossed module by PL1,
- ii)  $L$  has zero multiplication by PL2,
- iii)  $\partial M$  has zero action on  $L$  by PL4 and axiom PL5 is omitted.

The above remarks are known for 2-crossed modules of groups. These are handled in

book of Porter in [11].

### Examples of 2-crossed modules

1. Let  $(M, P, \partial)$  be a pre-crossed module. Then  $\{\ker \partial, M, P, i, \partial\}$  is a 2-crossed module with

the Peiffer lifting  $\{m_0, m_1\} = m_0 m_1 - \partial(m_1) \cdot m_0$ , for  $m_0, m_1 \in M$ .

PL1)  $i(\{m_0, m_1\}) = \{m_0, m_1\} = m_0 m_1 - \partial(m_1) \cdot m_0$ , for  $m_0, m_1 \in M$

PL2)  $\{i(x), i(y)\} = \{x, y\} = x y - \partial y \cdot x = x y - 0 \cdot x = x y$ , for  $x, y \in \ker \partial$ ,

PL3)  $\{m_0, m_1 m_2\} = m_0(m_1 m_2) - \partial(m_1 m_2) \cdot m_0$

$$\begin{aligned}
 &= (m_0m_1)m_2 - \partial(m_2) \cdot (m_0m_1) + \partial(m_2) \\
 &\quad \cdot (m_0m_1) - (\partial(m_2)\partial(m_1)) \cdot m_0 \\
 &= (m_0m_1)m_2 - \partial(m_2) \cdot (m_0m_1) + \partial(m_2) \\
 &\quad \cdot (m_0m_1) - \partial(m_2) \cdot (\partial(m_1) \cdot m_0) \\
 &= (m_0m_1)m_2 - \partial(m_2) \cdot (m_0m_1) + \partial(m_2) \cdot (m_0m_1 \\
 &\quad - \partial(m_1) \cdot m_0) \\
 &= \{m_0m_1, m_2\} + \partial(m_2) \cdot \{m_0, m_1\}, \text{ for } m_0, m_1, m_2 \\
 &\quad \in M
 \end{aligned}$$

$$\begin{aligned}
 PL4) \{m, i(x)\} &= \{m, x\} = mx - \partial(x) \cdot m \\
 &= mx - 0 \cdot m = mx = m \cdot x
 \end{aligned}$$

$$\begin{aligned}
 \{i(x), m\} &= \{x, m\} = xm - \partial(m) \cdot x \\
 &= m \cdot x - \partial(m) \cdot x, \text{ for } x \in \ker \partial, m \\
 &\in M
 \end{aligned}$$

$$\begin{aligned}
 PL5) p \cdot \{m_0, m_1\} &= p \cdot (m_0m_1 - \partial(m_1) \cdot m_0) \\
 &= p \cdot (m_0m_1) - p \cdot (\partial(m_1) \cdot m_0) \\
 &= p \cdot (m_0m_1) - (p\partial(m_1)) \cdot m_0 \\
 &= m_0(p \cdot m_1) - \partial(p \cdot m_1) \cdot m_0 \\
 &= \{m_0, p \cdot m_1\} \text{ and } p \cdot \{m_0, m_1\} \\
 &= \{p \cdot m_0, m_1\} \text{ for } m_0, m_1 \in M, p \\
 &\in P.
 \end{aligned}$$

2. Let  $\partial : M \rightarrow P$  be a pre-crossed module. We define  $\langle M, M \rangle$  as the ideal of  $M$  which is

generated by the elements, called Peiffer commutators,

$$\langle m, m' \rangle = mm' - \partial m' \cdot m$$

for all  $m, m' \in M$ . Since

$$\begin{aligned} n \langle m, m' \rangle &= n(mm' - \partial(m') \cdot m) \\ &= n(mm') - n(\partial(m') \cdot m) \\ &= (nm)m' - \partial(m') \cdot (nm) \in \langle M, M \rangle, \end{aligned}$$

$\langle M, M \rangle$  is an ideal of  $M$ , called by Peiffer ideal. If all Peiffer commutators are trivial,

$mm' - \partial m' \cdot m = \langle m, m' \rangle = 0$ , then we get  $\partial m' \cdot m = mm'$ .

Thus, the pre-crossed modules in which all Peiffer commutators are trivial are precisely

the crossed modules. Thus the category of crossed modules is the full subcategory of the

category of pre-crossed modules whose objects are crossed modules. Also, we can define

$\{(M, M), M, P, i, \partial\}$  as a 2-crossed module with the Peiffer lifting  $\{m, m'\} = \langle m, m' \rangle$ .

$$\begin{aligned} PL1) i(\{m_0, m_1\}) &= \{m_0, m_1\} = \langle m_0, m_1 \rangle \\ &= m_0 m_1 - \partial(m_1) \cdot m_0, \text{ for } m_0, m_1 \\ &\in M \end{aligned}$$

$$\begin{aligned}
 PL2) \{i(\langle m_0, m_1 \rangle), i(\langle m_2, m_3 \rangle)\} &= \{\langle m_0, m_1 \rangle, \langle m_2, m_3 \rangle\} \\
 &= \langle \langle m_0, m_1 \rangle, \langle m_2, m_3 \rangle \rangle \\
 &= \langle m_0, m_1 \rangle \langle m_2, m_3 \rangle - \partial(\langle m_2, m_3 \rangle) \cdot \langle m_0, m_1 \rangle \\
 &= \langle m_0, m_1 \rangle \langle m_2, m_3 \rangle - \partial(m_2 m_3 - \partial(m_3) \cdot m_2) \cdot \langle m_0, m_1 \rangle \\
 &= \langle m_0, m_1 \rangle \langle m_2, m_3 \rangle - (\partial(m_2 m_3) - \partial(\partial(m_3) \cdot m_2)) \cdot \langle m_0, m_1 \rangle \\
 &= \langle m_0, m_1 \rangle \langle m_2, m_3 \rangle - (\partial(m_2) \partial(m_3)) - \partial(m_3) \cdot \partial(m_2) \cdot \langle m_0, m_1 \rangle \\
 &= \langle m_0, m_1 \rangle \langle m_2, m_3 \rangle - 0 \cdot \langle m_0, m_1 \rangle \\
 &= \langle m_0, m_1 \rangle \langle m_2, m_3 \rangle \text{ for } \langle m_0, m_1 \rangle, \langle m_2, m_3 \rangle \in \langle M, M \rangle
 \end{aligned}$$

PL3) see example 1, PL3.

PL4)

$$\begin{aligned}
 \{m_0, & \quad = \langle m_0, \langle m_1, m_2 \rangle \rangle \\
 i(\langle & \\
 m_1, m_2 \rangle)\} & \\
 &= m_0 \langle m_1, m_2 \rangle - \partial(\langle \\
 m_1, m_2 \rangle) \cdot m_0 \\
 &= m_0 \langle m_1, m_2 \rangle \\
 &- \partial(m_1 m_2 - \partial(m_2) \cdot m_1) \cdot m_0 \\
 &= m_0 \langle m_1, m_2 \rangle \\
 &- (\partial(m_1 m_2) - \partial(\partial(m_2) \cdot m_1)) \cdot \\
 m_0 \\
 &= m_0 \langle m_1, m_2 \rangle \\
 &- (\partial(m_1 m_2) - \partial(m_2) \cdot \partial(m_1)) \cdot \\
 m_0 \\
 &= m_0 \langle m_1, m_2 \rangle \\
 &- (\partial(m_1) \partial(m_2) -
 \end{aligned}$$



$$\begin{aligned}
 & \partial(m_2)\partial(m_1)) \cdot m_0 \\
 & = m_0 < m_1, m_2 > - 0 \cdot m_0 \\
 & = m_0 < m_1, m_2 > = m_0 \cdot < \\
 & m_1, m_2 > \\
 & \{i(< \qquad \qquad \qquad = << m_0, m_1 >, m_2 > \\
 & m_0, m_1 >, m_2)\} \\
 & \qquad \qquad \qquad = < m_0, m_1 > m_2 - \\
 & \partial(m_2) \cdot < m_0, m_1 > \\
 & = m_2 < m_0, m_1 > - \partial(m_2) \cdot \\
 & < m_0, m_1 > \\
 & \qquad \qquad \qquad = m_2 \cdot < m_0, m_1 > \\
 & - \partial(m_2) \cdot < m_0, m_1 >
 \end{aligned}$$

PL5) see example 1, PL5.

2. Any crossed module gives a 2-crossed module.

If  $(M, P, \partial)$  is a crossed module, we get

$\{0, M, P, 0, \partial\}$  as a 2-crossed module with trivial Peiffer lifting.

### ***Functorial Relations with Some Other Categories***

By the definition of crossed and 2-crossed module there exists the following functors:

1.  $\gamma, \mu : \mathbf{k}\text{-Alg} \rightarrow \mathbf{XMod}$  given by  $\gamma(A) = (A, A, id_A)$ ,  $\gamma(f) = (f, f)$  and  $\mu(A) =$

$(0, A, 0)$ ,  $\mu(f) = (0, f)$  for any  $\mathbf{k}$ -algebra morphism  $f: A \rightarrow B$ .

2.  $\delta: \mathbf{XMod} \rightarrow \mathbf{k}\text{-Alg}$  given by  $\delta(C, R, \partial) = R$ , and  $\delta(\theta, \vartheta) = \vartheta: R \rightarrow R'$  for any

crossed module morphism  $(\theta, \vartheta): (C, R, \partial) \rightarrow (C', R', \partial')$ .

3.  $\rho: R\text{-Mod} \rightarrow \mathbf{XMod}/R$ , given by  $\rho(M) = (M, 0, 0)$  and for any morphism of

$R$ -modules  $h: M \rightarrow N$ ,  $\rho(h) = (h, 0): (M, 0, 0) \rightarrow (N, 0, 0)$  as a morphism in  $\mathbf{XMod}/R$ .

4.  $\eta: \mathbf{XMod}/R \rightarrow R\text{-Mod}$ , given by  $\eta(C, R, \partial) = \ker \partial$ , and  $\eta(\theta, \vartheta) = \theta|_{\ker \partial}: \ker \partial \rightarrow \ker \partial'$  by for any crossed module morphism  $(\theta, \vartheta): (C, R, \partial) \rightarrow (C', R', \partial')$  where the action of  $R$  on

$\ker \partial$  is well-defined because of  $\partial(r \cdot x) = r\partial(x) = r0 = 0$  for  $r \in R, x \in \ker \partial$  and since

$\partial'(\theta(x)) = \partial(x) = 0$ , we get  $\theta|_{\ker \partial}(x) = \theta(x) \in \ker \partial'$ .

5.  $\lambda: X\text{Mod} \rightarrow \mathbf{k}\text{-Alg}$ , given by  $\lambda(C, R, \partial) = R/\partial C$ , and  $\lambda(\theta, v) = v': R/\partial C \rightarrow R/\partial C'$  defined by  $v'(r + \partial C) = v(r) + \partial C'$  for any crossed module morphism  $(\theta, v): (C, R, \partial) \rightarrow (C', R', \partial')$ .

6. The Skeleton functor  $Sk : \text{P}X\text{Mod} \rightarrow X_2\text{Mod}$ , given by  $Sk(M, P, \partial) = \{\langle M, M \rangle, M, P, i, \partial\}$

for any pre-crossed module morphism  $(f_1, f_0): (M, P, \partial) \rightarrow (M', P', \partial')$ ,

$$Sk(f_1, f_0) = (f_2, f_1, f_0) : \{\langle M, M \rangle, M, P, i, \partial\} \rightarrow \{\langle M', M' \rangle, M', P', i', \partial'\}$$

where  $f_2(\langle m_1, m_2 \rangle) = \langle f_1(m_1), f_1(m_2) \rangle$ .

Since

$$\begin{aligned} f_2(\langle m_1, m_2 \rangle) &= \langle f_1(m_1), f_1(m_2) \rangle \\ &= f_1(m_1) f_1(m_2) - \partial'(f_1(m_2)) \cdot f_1(m_1) \\ &= f_1(m_1) f_1(m_2) - f_0(\partial(m_2)) \cdot f_1(m_1) \\ &= f_1(m_1) f_1(m_2) - f_1(\partial(m_2)) \cdot m_1 \\ &= f_1(m_1 m_2 - \partial(m_2)) \cdot m_1 \\ &= f_1(\langle m_1, m_2 \rangle) \end{aligned}$$

and  $f_1$  is a  $\mathbf{k}$ -algebra morphism, we get that  $f_2$  is a  $\mathbf{k}$ -algebra morphism.

Since

$$\begin{aligned} p \cdot \langle m_1, m_2 \rangle &= p \cdot (m_1 m_2 - \partial(m_2)) \cdot m_1 \\ &= m_1 (p \cdot m_2) - p \cdot (\partial(m_2)) \cdot m_1 \\ &= m_1 (p \cdot m_2) - \partial(p \cdot m_2) \cdot m_1 \\ &= \langle m_1, p \cdot m_2 \rangle, \end{aligned}$$

we have

$$\begin{aligned}
 f_2(p \cdot \langle m_1, m_2 \rangle) &= f_2(\langle m_1, p \cdot m_2 \rangle) \\
 &= \langle f_1(m_1), f_1(p \cdot m_2) \rangle \\
 &= \langle f_1(m_1), f_0(p) \cdot f_1(m_2) \rangle \\
 &= f_0(p) \cdot \langle f_1(m_1), f_1(m_2) \rangle \\
 &= f_0(p) \cdot f_2(\langle m_1, m_2 \rangle).
 \end{aligned}$$

Since  $(f_1, f_0)$  is a precrossed module morphism,

$$\partial' f_1 = f_0 \partial$$

and also

$$\begin{aligned}
 i' f_2(\langle m_1, m_2 \rangle) &= i'(\langle f_1(m_1), f_1(m_2) \rangle) \\
 &= \langle f_1(m_1), f_1(m_2) \rangle \\
 &= f_1(\langle m_1, m_2 \rangle) \\
 &= f_1 i(\langle m_1, m_2 \rangle),
 \end{aligned}$$

$$\begin{aligned}
 \{ \} ' f_1 \times f_1(m_1, m_2) &= \{ \} '(f_1(m_1), f_1(m_2)) \\
 &= \{ f_1(m_1), f_1(m_2) \}' \\
 &= \langle f_1(m_1), f_1(m_2) \rangle \\
 &= f_2(\langle m_1, m_2 \rangle) \\
 &= f_2 \{ m_1, m_2 \} \\
 &= f_2 \{ \} (m_1, m_2).
 \end{aligned}$$

So  $(f_2, f_1, f_0)$  is a 2-crossed module morphism.

7. The Truncation functor  $Tr : X_2\text{Mod} \rightarrow \text{PXMod}$  given by

$$Tr\{L, M, P, \partial_2, \partial_1\} = (M, P, \partial_1)$$

and for any 2-crossed module morphism  $(f_2, f_1, f_0) : \{L, M, P, \partial_1, \partial_2\} \rightarrow \{L', M', P', \partial_1', \partial_2'\}$ ,

$$Tr (f_2, f_1, f_0) = (f_1, f_0).$$

**Proposition 1** *The functor  $\delta: XMod \rightarrow k-Alg$  which is given by  $\delta (C,R, \partial) = R$  has a left*

$$adjoint  $\mu : k-Alg \rightarrow XMod, \mu(A) = (0,A, 0).$$$

**Proposition 2** *The functor  $\delta: XMod \rightarrow k-Alg$  which is given by  $\delta (C,R, \partial) = R$  has a right*

$$adjoint  $\gamma: k-Alg \rightarrow XMod, \gamma(M) = (M,M, id_M)$$$

**Proof.** Let  $\gamma: k-Alg \rightarrow XMod$ , be defined by  $\gamma(M) = (M,M, id_M)$ . Then any object

$(C,R, \partial) \in Obj (XMod)$  and  $M \in Obj (k-Alg)$  define the morphism

$$\Gamma: k-Alg (\delta(C, R, \partial), M) \rightarrow XMod((C, R, \partial), \gamma(M))$$

as follows, if  $\theta : R \rightarrow M$  is a  $k$ -algebra morphism, then  $\Gamma (\theta) = (\theta \partial, \theta) : (C,R, \partial) \rightarrow$

$(M,M, id_M)$  is a crossed module morphism. Conversely, for any crossed module morphism

$$(\theta, \emptyset): (C,R, \partial) \rightarrow (M,M, id_M)$$
 we define

$$\Omega: XMod((C,R,\partial),\gamma(M)) \rightarrow k-Alg(\delta(C,R,\partial),M), \Omega((\theta, \emptyset)) = \theta: R \rightarrow M$$

which is a  $k$ -algebra morphism. We get  $\Omega\Gamma(\theta)=\Omega(\theta\partial,\theta)=\theta=id(\theta)$  and  $\Gamma\Omega((\theta, \emptyset))=\Gamma(\emptyset)=(\emptyset\partial,\emptyset)=(\theta, \emptyset)=id((\theta, \emptyset))$ . Thus  $\Gamma$  is a bijection. Also, for any crossed module morphism  $f:(C',R',\partial')\rightarrow(C,R,\partial)$  and algebra morphism  $k : M \rightarrow M'$  the following

diagrams are commutative and  $\delta^{C\mathcal{M}}$  is natural in  $(C,R, \partial)$  and  $M$ , respectively.

$$\begin{array}{ccc}
 k - \text{Alg}(\delta(C, R, \partial), M) & \xrightarrow{\delta^{C, \mathcal{M}}} & \text{XMod}((C, R, \partial), \gamma(M)) \\
 \delta(f)^* = -\circ\delta(f) \downarrow & & -\circ f = f^* \downarrow \\
 k - \text{Alg}(\delta(C', R', \partial'), M) & \xrightarrow{\delta^{C', \mathcal{M}}} & \text{XMod}((C', R', \partial'), \gamma(M)) \\
 \\
 k - \text{Alg}(\delta(C, R, \partial), M) & \xrightarrow{\delta^{C, \mathcal{M}}} & \text{XMod}((C, R, \partial), \gamma(M)) \\
 k\circ = k^* \downarrow & & \gamma(k)\circ \downarrow \\
 k - \text{Alg}(\delta(C, R, \partial), M') & \xrightarrow{\delta^{C, \mathcal{M}'}} & \text{XMod}((C, R, \partial), \gamma(M'))
 \end{array}$$

**Proposition 3** *The functor  $\alpha: \text{XMod} \rightarrow \text{X}_2\text{Mod}$  defined by  $\alpha(C, R, \partial) = \{0, C, R, 0, \partial\}$  is a right adjoint for the functor  $\beta: \text{X}_2\text{Mod} \rightarrow \text{XMod}$  given by  $\beta\{L, M, P, \partial_2, \partial_1\} = (M/\text{Im}\partial_2, P, \partial_1)$  where  $\text{Im}\partial_2$  is an ideal of  $M$ .*

**Proof.** Since  $\partial_2(m \cdot 1) = m\partial_2(1) \in \text{Im}\partial_2$ ,  $\text{Im}\partial_2$  is an ideal of  $M$  and  $\overline{\partial_1}(m + \text{Im}\partial_2) = \partial_1(m)$

is well-defined because of  $\partial_2(\partial_1(L))= 0$ , also the axioms of crossed modules are satisfied:

$$\begin{aligned} \overline{\partial_1}(p \cdot (m + \text{Im } \partial_2)) &= \overline{\partial_1}((p \cdot m) + \text{Im } \partial_2) \\ &= \partial_1(p \cdot m) \\ &= p \partial_1(m) \\ &= p \overline{\partial_1}(m + \text{Im } \partial_2) \end{aligned}$$

$$\begin{aligned} \overline{\partial_1}(m + \text{Im } \partial_2) \cdot (m' + \text{Im } \partial_2) &= \partial_1(m) \cdot (m' + \text{Im } \partial_2) \\ &= (\partial_1(m) \cdot m') + \text{Im } \partial_2 \\ &= mm' - \partial_2\{m, m'\} + \text{Im } \partial_2 \\ &= mm' + \text{Im } \partial_2. \end{aligned}$$

For any object  $\mathcal{L} = \{L, M, P, \partial_2, \partial_1\} \in \text{Obj}(X_2 \text{Mod})$  and  $\mathcal{C} = (C, R, \partial) \in \text{Obj}(X \text{Mod})$  we

define the morphism

$\Phi$ :

$$X \text{Mod}(\beta(\{L, M, P, \partial_2, \partial_1\}), (C, R, \partial)) \rightarrow X_2 \text{Mod}(\{L, M, P, \partial_2, \partial_1\}, \alpha(C, R, \partial))$$

as follows, if  $(f_1, f_0): (M/\text{Im } \partial_2, P, \partial_1) \rightarrow (C, R, \partial)$  is a crossed module morphism, then

$\Phi(f_1, f_0) = (0, f_1 q, f_0)$  is a 2-crossed module morphism. Conversely for any 2-crossed module

morphism  $(0, f_1, f_0): \{L, M, P, \partial_2, \partial_1\} \rightarrow \{0, C, R, 0, \partial\}$  we define

$\Psi: X_2\text{Mod} (\{L, M, P, \partial_2, \partial_1\}, \alpha(C, R, \partial)) \rightarrow X\text{Mod} (\beta(\{L, M, P, \partial_2, \partial_1\}), (C, R, \partial))$

As  $\Psi((0, f_1, f_0)) = (f_1^*, f_0): (M/Im\partial_2, P, \partial_1) \rightarrow (C, R, \partial)$  where  $f_1^*, \partial_1^*$  are well-defined because of  $f_1(\partial_2(L))=0$  and  $\partial_1(\partial_2(L)) = 0$ , respectively and  $f_1^*(m + Im\partial_2) = f_1(m)$ . We see that

$$\begin{aligned} (\Phi\Psi)((0, f_1, f_0)) &= \Phi(f_1^*, f_0) = (0, \\ f_1^*q, f_0) &= (0, f_1, f_0) = \text{id}(0, f_1, f_0) \end{aligned}$$

and

$$\Psi\Phi(f_1, f_0) = \Psi(0, f_1q, f_0) = (t, f_0) = (f_1, f_0) = \text{id}(f_1, f_0)$$

where  $t(m + Im\partial_2) = f_1q(m) = f_1(m + Im\partial_2)$  for all  $m + Im\partial_2 \in M/Im\partial_2$ . Thus  $\Phi$  is a

bijection.

Moreover; for crossed module morphisms  $\mu = (\mu_1, \mu_0) : \mathcal{C} \rightarrow \mathcal{C}'$  and  $(\bar{f}, f_0) : \beta(L) \rightarrow \mathcal{C}$  since

$$\begin{aligned} \alpha(\mu)^* \beta^{\mathcal{L}\mathcal{C}}(\bar{f}, f_0) &= \alpha(\mu)^*(0, \bar{f}q, f_0) \\ &= (0, \mu_1\bar{f}q, \mu_0f_0) \\ &= \beta^{\mathcal{L}\mathcal{C}'}(\mu_1\bar{f}, \mu_0f_0) \\ &= \beta^{\mathcal{L}\mathcal{C}'}\mu^*(\bar{f}, f_0), \end{aligned}$$

the following diagram is commutative and  $\beta^{\mathcal{L}\mathcal{C}}$  is natural in  $\mathcal{C}$ ,



$$\begin{array}{ccc}
 \text{XMod}(\beta\mathcal{L}, \mathcal{C}) & \xrightarrow{\beta^{\mathcal{L}\mathcal{C}}} & \text{X}_2\text{Mod}(\mathcal{L}, \alpha\mathcal{C}) \\
 \downarrow \mu \circ - = \mu^* & & \downarrow \alpha(\mu) \circ - = \alpha(\mu)^* \\
 \text{X}_2\text{Mod}(\beta\mathcal{L}, \mathcal{C}') & \xrightarrow{\beta^{\mathcal{L}\mathcal{C}'}} & \text{XMod}(\mathcal{L}, \alpha\mathcal{C}'),
 \end{array}$$

for any 2-crossed module morphism  $(\epsilon_2, \epsilon_1, \epsilon_0): \mathcal{L}' \rightarrow \mathcal{L}$ , crossed module morphism

$(k, f_0) : \beta(\mathcal{L}) \rightarrow \mathcal{C}$ , the quotient maps  $q : M \rightarrow M/\text{Im}\partial_2$ ,  $q' : M' \rightarrow M'/\text{Im}\partial_2'$

and  $\lambda: M'/\text{Im}\partial_2' \rightarrow M/\text{Im}\partial_2$  defined by  $\lambda(m' + \text{Im}\partial_2') = \epsilon_1(m') + \text{Im}\partial_2$ , since

$$\epsilon^* \beta^{\mathcal{L}\mathcal{C}}(k, f_0) = \epsilon^*(0, kq, f_0) = (0, kq\epsilon_1, f_0\epsilon_0),$$

$$\beta^{\mathcal{L}'\mathcal{C}} \beta(\epsilon)^*(k, f_0) = \beta^{\mathcal{L}'\mathcal{C}}(k\lambda, f_0\epsilon_0) = (0, k\lambda q', f_0\epsilon_0)$$

and

$$\begin{aligned}
 k\lambda q'(m') &= k\lambda(m' + \text{Im}\partial_2') \\
 &= k(\epsilon_1(m') + \text{Im}\partial_2) \\
 &= kq\epsilon_1(m')
 \end{aligned}$$

the following diagram

$$\begin{array}{ccc}
 X_2\text{Mod}(\beta\mathcal{L}, \mathcal{C}) & \xrightarrow{\beta^{\alpha\mathcal{C}}} & X\text{Mod}(\mathcal{L}, \alpha\mathcal{C}) \\
 \downarrow \beta(\epsilon)^* = -\circ\beta(\epsilon) & & \downarrow -\circ\epsilon = \epsilon^* \\
 X_2\text{Mod}(\beta\mathcal{L}', \mathcal{C}) & \xrightarrow{\beta^{\alpha'\mathcal{C}}} & X\text{Mod}(\mathcal{L}', \alpha\mathcal{C})
 \end{array}$$

is commutative and  $\beta^{\mathcal{L}\mathcal{C}}$  is natural in  $\mathcal{L}$ .

**Proposition 4** Given  $X_2\text{Mod} \xrightleftharpoons[\alpha]{\beta} X\text{Mod} \xrightleftharpoons[\gamma]{\delta} k\text{-Alg}$   
 adjoint functors as defined above. Then

$(\delta^\circ\beta, \alpha^\circ\gamma)$  is a pair of adjoint functors.

**Proof.** Since  $(\beta, \alpha)$  and  $(\delta, \gamma)$  are two adjoint pairs, we have

$$k\text{-Alg}(\delta(C, R, \partial), R) \simeq X\text{Mod}((C, R, \partial), \gamma(R))$$

and

$$\begin{aligned}
 X\text{Mod}(\beta\{L, M, P, \partial_2, \partial_1\}, (C', R', \partial')) &\simeq \\
 X_2\text{Mod}(\{L, M, P, \partial_2, \partial_1\}, \alpha((C', R', \partial'))) &
 \end{aligned}$$

for  $\{L, M, P, \partial_2, \partial_1\} \in X_2\text{Mod}$ ,  $(C, R, \partial), (C', R', \partial') \in X\text{Mod}$  and  $R \in k\text{-Alg}$ . Setting  $(C, R, \partial) := \beta\{L, M, P, \partial_2, \partial_1\}$  and  $(C', R', \partial') := \gamma(R)$ , we get

$$\begin{aligned} X_2 \text{Mod} (\{L, M, P, \partial_2, \partial_1\}, \alpha(\gamma(R))) &\simeq X \text{Mod} (\beta\{L, M, P, \partial_2, \partial_1\}, \gamma(R)) \\ &\simeq k - \text{Alg}((\delta(\beta\{L, M, P, \partial_2, \partial_1\}), R)). \end{aligned}$$

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